THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 53, PART 6 1 November 1941 No. 300

CHARLES CHREE AND HIS WORK ON GEOMAGNETISM

BY S. CHAPMAN, M.A., D.Sc., F.R.S.

Part I of the first Charles Chree Address, delivered 25 July 1941

ABSTRACT. After having gained distinction as an authority on the mathematical theory of elasticity, Charles Chree's interests were turned to the study of geomagnetism by his appointment, at the age of 33, as Superintendent of the Kew Observatory. There he gained an enduring place in the list of distinguished British contributors to geomagnetism. His work is briefly reviewed, particularly in its relation to that of his contemporary fellow-workers in the subject. He did outstanding service, in addition to maintaining the regular activities of his observatory at a high level, in discussing the data obtained there and on four Antarctic expeditions. Though critical of current physical theories of the geomagnetic variations, and though he himself refrained from speculation on the physical causes of geomagnetic disturbance, his examination of the theoretical views of Maunder and Arrhenius, by the discussion of the magnetic data, led him to his finest achievements.

§1. THE SUPERINTENDENT OF KEW OBSERVATORY

CHARLES CHREE, a son of the manse, an Aberdonian scholar brilliant in classics as well as in science, distinguished later at Cambridge as an expert on the mathematical theory of elasticity, in 1893 took up new and very different labours as Superintendent of the magnetic and meteorological observatory at Kew. He held this position until 1925, three years before his death at the age of 68. When he retired he was president of both the international organizations concerned with geomagnetism.

§2. THE CHARLES CHREE MEDAL AND PRIZE

His sister, Miss Jessie Chree, who shared his home at Richmond, near Kew, has now founded the Charles Chree Medal and Prize, in remembrance of her distinguished brother, and to encourage the study of the science in which he found a happy life-work. The Physical Society, whose President he had been, agreed to administer her generous benefaction, and through Mrs. Mary Gillick's art the Society has a fine medal in its gift. I am privileged and proud to be the first recipient of this honour and gift.

§3. BRITISH CONTRIBUTIONS TO GEOMAGNETISM

It is among the glories of science that it offers scope for generous international co-operation: in the earth sciences, most of all, such co-operation is a necessity for progress, and Charles Chree took a notable part in it.

Yet each nation may feel pride in its own contributions to the common stock of knowledge, and in earth magnetism Britain has a distinguished record from the sixteenth century onward. Queen Elizabeth's Hydrographer, Robert Norman, was the first to measure the magnetic dip (1576); her physician, William Gilbert, wrote the first modern scientific treatise (1600), and the earth's magnetism was its subject; the Gresham professor, Henry Gellibrand, discovered the secular magnetic variation (1634); Edmund Halley, Oxford professor and, later, Astronomer Royal, made the first scientific voyages (1698–1700), and his object was to measure the magnetic declination over the North and South Atlantic Oceans; he also constructed the first magnetic chart (1701); George Graham, the London instrument maker, discovered the transient magnetic variations (1724).

In the nineteenth century George Biddel Airy, one of the greatest of the Astronomers Royal, founded and developed the magnetic observatory at Greenwich (1841). Edward Sabine fostered the British Colonial Magnetic Observatories, and discovered the 11-year sunspot cycle in geomagnetism (1841–8): like James Allan Broun at Trevandrum (1874), Sabine also made notable studies of the lunar geomagnetic tide. Balfour Stewart was the first to realize the existence of the ionosphere, and he proposed the dynamo theory of the daily magnetic variations (1882). Arthur Schuster applied Gauss's method of spherical harmonic analysis to these variations (1889), and later developed Stewart's dynamo theory (1908). Edward Walter Maunder made brilliant studies on the connection between the sun and magnetic storms (1904–16). Charles Chree, who was a contemporary of Schuster and Maunder, was in this notable succession, and his name will endure in the history of the subject.

§4. CHARLES CHREE AND HIS CONTEMPORARIES

Besides these three, the most distinguished contemporary workers on geomagnetism were Adolf Schmidt, the (German) director of the Potsdam Magnetic Observatory, a most learned, able and many-sided contributor to the subject; Louis A. Bauer, the (American) initiator and first director of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington; W. van Bemmelen, the (Dutch) director of the Batavia Observatory; and N. A. F. Moos, the (Indian) director of the Bombay Magnetic Observatory. Moos published a monumental study (1910) of the Bombay data, covering 60 years. The story of the work of these seven men would epitomize the progress of earth magnetic science during nearly half a century, though my list of the leaders of the science during the half century ending with Dr. Chree's death would need to be much lengthened if it were to include all the devoted and zealous men who, on land or

sea, in frequented or in remote regions, took part in the great and never-ending task of the world magnetic survey. Bauer viewed this task as a whole, and one main enterprise of his Department was to fill up the widest and most urgent gaps in the magnetic survey—mainly on sea, but also on land—that were not covered by existing official or private efforts.

The improvement of instruments and methods for magnetic measurement is an important aid to the world magnetic survey, and Bauer's Department, Schmidt and Schuster all made valuable contributions to this work. Chree at Kew Observatory assisted by standardizing magnetic instruments and by training many young men to use them, notably on polar and other scientific expeditions.

§ 5. MAGNETIC OBSERVATORIES

Apart from magnetic surveys, our knowledge of earth magnetism comes mainly from magnetic observatories, of which there are about seventy-five in the whole world, not well distributed. The accuracy of measurement attempted at a first-rate magnetic observatory is so high that the work of the director is exacting. Kew Observatory was a centre to which many others looked for the calibration of their instruments, and Chree maintained this high repute, even after disturbance due to London electric traction rendered Kew no longer a suitable site for a magnetic observatory. He also played a valuable part in the institution of a substitute observatory at Eskdalemuir, in the Scottish Lowlands, and later in the addition of another British observatory at Lerwick in Shetland.

The magnetic changes at an observatory must be recorded day by day if they are not to remain for ever unknown; but the photographic records form only the rawest material for geomagnetic science, and much measurement and computation is needed to make them available for discussion. Many observatories are content to publish their measured data, casting their bread, as it were, upon the waters; many years rather than days may elapse before it is found and used. Only a few observatory directors have the combination of time, energy, ability, and inward impulse to undertake the scientific discussion of their records; Charles Chree was eminent among these.

§ 6. CHARLES CHREE'S WORK AT KEW

His devotion to this work cannot be over-praised. When he became Superintendent, Kew Observatory was not a State institution; its financial position was precarious, with only a meagre endowment income; it was maintained partly by fees for testing and calibrating thermometers, watches and instruments numbering 20 000 a year. Yet although it was the central British meteorological observatory, as well as being a magnetic and electric observatory, its staff only numbered fifteen. Chree supervised its activities faithfully and well; moreover, he wrote on meteorology and atmospheric electricity, and in time became President of the Royal Meteorological Society; but he was most

attracted by the magnetic work, and gave close attention to the operation of the magnetic recorders, the calibration of instruments for other observatories, and the training of magnetic observers, which he himself undertook; in addition he wrote about seventy papers and memoirs on terrestrial magnetism, including many of considerable length. He devoted immense effort and time to the discussion not only of the Kew magnetic data, but also of records obtained at the Falmouth and Eskdalemuir observatories, and at the magnetic stations occupied by four Antarctic expeditions, including those of Scott and Mawson. A great deal of this work was done at the sacrifice of his own leisure time. In labours of this kind probably only one other observatory director, Moos of Bombay, can be compared with him. None but the strongest devotion to the science can impel a man freely to take up such a burden; yet only by such work can the great mass of data annually gathered in the magnetic observatories of the world be made to disclose the laws and regularities, and also the statistics of variability, in the phenomena, like an elixir distilled from a great volume of matter.

Chree brought to this work, besides untiring industry and devotion, a rare quality of cautious and open-minded criticism, and scrupulous exactitude in stating his conclusions. It was foreign to his nature to slur over evidence that precluded simple general statements, however attractive such simplicity might be. His attention to detail was meticulous, and every facet of the truth must be expounded. Doubtless his retentive mind could survey the whole terrain that he thus explored, but others less familiar with it often did not receive the help they needed to distinguish the more from the less important. The art of emphasis, of displaying his wares, made little appeal to his serious austere mind. For whoever would receive it he provided solid fare of fact and knowledge, but with little dressing of the dish, or pleasing of the eye or palate, except (and it was a notable exception) that his speech, and at times his writings, were enlivened by a pleasant characteristic humour.

§7. THE MANY-SIDED NEEDS OF EARTH MAGNETIC SCIENCE

Geomagnetism has need of men of widely different gifts: of the organizer and administrator; of the man of affairs, who can influence governments or private benefactors to provide resources for the science; of the navigator and explorer; of the instrument designer and maker; of the skilled, patient and resourceful observer, often far from social and technical centres; of the faithful computer, zealous for detailed accuracy; of the investigator, theorist and writer.

Louis Bauer was a man of fiery enthusiasm and dynamic energy, who took wide views and realized great projects; he has left a deep and permanent impress on the science. But his sanguine eagerness to reach conclusions, where sometimes judgement was better held in suspense, met a useful corrective in Chree's critical detachment, which later administered some cooling draughts to my own

youthful optimism. Arthur Schuster, though not Chree's equal as a mathematician, nor with a tithe of his detailed knowledge of geomagnetism, could make brilliant sorties into the subject, leading to striking theoretical conclusions or illuminations; by labours trifling compared with those expended by Chree, he could lift some part of the subject on to a new plane. Walter Maunder brought to geomagnetism a daily familiarity with the changes on the sun's surface, as well as with the Greenwich magnetic records; this enabled him to perceive some of the remarkable associations between the sun and magnetic storms; and though he had little mathematics, he possessed a gift of graphical representation, a skill in showmanship, that was of great scientific value. His work stimulated Chree to some of his finest achievements.

§8. CHREE'S VARIED MAGNETIC WORK

In the time available to me it is impossible to describe all Dr. Chree's researches, and what I can say will doubtless be supplemented in future addresses on such occasions as this. Among the topics to which he devoted much attention, but which I will only mention, are magnetic activity and its quantitative estimation by character figures or otherwise; the non-cyclic variation or recovery from magnetic disturbance, which he and van Bemmelen discovered independently in 1895; the Fourier coefficients of the daily magnetic variation, and their dependence on the season and sunspot number; the association between aurorae and magnetic storms and sunspots; eclipse effects on the earth's magnetic field; the degree of world-wide simultaneity of sudden commencements of magnetic storms; the differences between the magnetic changes in the polar and temperate regions; the secular variation of the earth's field; and the relation (or lack of relation) between geomagnetic and meteorological or atmospheric electric variations.

I will speak mainly (in Part II of this Address) on some aspects of his many studies of the *time* relations between the sun and geomagnetic disturbance; but first I will indicate something of his general attitude to geomagnetic research.

§9. CHREE IN RELATION TO GEOMAGNETIC THEORIES AND SPECULATIONS

Dr. Chree once told me that in his youth he was regarded as dangerously speculative; in his maturity, however, he had outgrown this attitude, and, unlike Schuster, Maunder and Bauer, he avoided hypothesis and speculation. This, he declared, was due to no lack of curiosity as to the causes of things, but to a belief that at the present stage theorizing is less likely to be of substantial advantage than the extension of positive knowledge; and also because physicists may still be unaware of facts essential to geomagnetic theories. He held that a theory was not necessary as a guide to research. Certainly his work and his discoveries showed that it was not necessary for him to *hold* any theoretical views; but it seems equally clear that some theories that were advanced by others did to a considerable extent determine the nature of his studies.

In support of this conclusion I may refer to his valuable critical review (1905) of Maunder's work (1904) on the recurrence of magnetic storms. Maunder was led to the view that: "Our magnetic disturbances have their origin in the sun. The solar action which gives rise to them does not act equally in all directions, but along narrow, well-defined streams, not necessarily truly radial. These streams arise from active areas of limited extent. These active areas are not only the source of our magnetic disturbances, but are also the seats of the function of sunspots . . . . But these areas can be magnetically active both before a spot has formed and after it has disappeared."

This was Chree's final comment in his review:—"It only remains to say that whilst in the opinion of the writer further investigation is required to justify the final acceptance of any of Mr. Maunder's views, his paper is a most important one. The theory advanced hardly touches the physical side of the problem, but it is clear and definite so far as it goes, and is obscured by no mystifications of language. It is certainly incomplete, and may be wholly erroneous, but it is exceedingly suggestive, and indicates a number of lines of research which can hardly fail to lead to valuable results."

Another theory that influenced the direction of Chree's researches was one advanced by Arrhenius and others (apparently first mentioned by Chree in his writings in 1908) that the solar influence responsible for magnetic disturbance is propagated to the earth in two or three days; Birkeland, on the other hand, thought that the solar agent consisted of electrons travelling with almost the speed of light, and therefore reaching the earth in less than an hour.

To me it seems that Dr. Chree's work became far more interesting, valuable and original from the time, in 1908, when he began to test these theories of Maunder and Arrhenius.

Previously his main work had been concerned with the changes in the *type* and *intensity* of the daily magnetic variations in the course of the seasons and the 11-year sunspot cycle, and as modified by magnetic disturbance; from 1908 onwards much of his work was concerned with the *time relationships* of magnetic phenomena *inter se*, and between them and solar phenomena.

REFERENCES

A short account of the history of the growth of knowledge on geomagnetism, together with many bibliographical references, is given in *Geomagnetism*, by S. Chapman and J. Bartels, 1940 (Oxford University Press); another extensive bibliography of the subject is given in *Terrestrial Magnetism and Electricity*, 1939, edited by J. A. Fleming (McGraw-Hill). The following are the references to the papers specially mentioned in § 9:—

Chree, C., 1905. *Terr. Mag.* **10**, 14.

Maunder, E. W., 1904. *Mon. Not. Roy. Astr. Soc.* **64**, 205 (for further references see *Geomagnetism*, p. 974).

GEOMAGNETIC TIME-RELATIONSHIPS

By S. CHAPMAN, M.A., D.Sc., F.R.S.

Part II of the first Charles Chree Address, delivered 25 *July* 1941

ABSTRACT. On the average, on days of notable geomagnetic disturbance, the sunspottedness is declining from a maximum value attained two or three days earlier; this is proved most directly by the *method of superposed epochs*, devised by Charles Chree for the study of geomagnetic time-relationships. The method lends itself well to graphical illustration. By the same method Chree gave the most convincing demonstration of the existence and interval of the 27-day recurrence tendency, which Maunder had previously demonstrated by means of a time-pattern of magnetic storms, and interpreted in terms of the solar rotation. The time-pattern method of studying geomagnetic disturbance and sunspottedness (either alone or comparatively) was developed further by Chree and Stagg and perfected by Bartels. Chree's method of superposed epochs has recently been successfully applied to cosmic-ray data.

§1. MAGNETIC DISTURBANCE AND CONTEMPORANEOUS SUNSPOTS

In his work on the time-relationships of geomagnetic disturbance, Dr. Chree first examined whether there was any clear relation between the *simultaneous* intensities of solar and magnetic disturbance. In each month throughout 11 years he divided the days into three nearly equal groups according to their *sunspot area*. He then examined the proportion of magnetically quiet days that fell in each group, and likewise the proportion of magnetically disturbed days; the quiet days, then chosen by the Astronomer Royal, numbered five per month, and the disturbed days, selected by Chree himself, numbered on an average 19 per year. The results were as follows (Chree, 1908, p. 236):—

	Days of greatest spot area	Days of intermediate spot area	Days of least spot area
Mean sunspot area (Greenwich units)	1626	—	312
Percentage of quiet days	31·7	34·5	33·8
Percentage of disturbed days	35·6	34·0	30·4

This showed that magnetically quiet days were slightly less numerous when the sunpot area was large than when it was small, and conversely for disturbed days. But the differences of frequency are very slight. Chree therefore said: "We must, I think, conclude that these results are incompatible with any theory which regards magnetic disturbance as dependent directly *in any large degree* on the *simultaneous* extent of the projected sunspot area." He thought, however, that the results "might be easily reconciled with theories such as that of Arrhenius", which postulated a lag of two or three days between the solar cause and the geomagnetic disturbance.

§ 2. DOES GEOMAGNETIC DISTURBANCE LAG BEHIND SOLAR ACTIVITY?

To study this possibility Chree devised and introduced, I believe for the first time, a method of great value and power, which in a recent book I have called the method of *superposed epochs*. He examined how, on the average, the sunspot area varied during periods of four days, each period ending with a day of marked magnetic character, quiet or disturbed, and including the three preceding days. For this purpose he selected the ten days per month of largest daily range in Kew declination, and the ten per month of smallest daily range. The four-day period ending with each selected day of large range gave him four sunspot areas, which he wrote down in a row. Having done this for all these magnetically disturbed days, he averaged the values in each column. He thus got the average sunspot area for a number of superposed epochs of magnetic disturbance, and for the three days preceding such epochs. He did the same for the quiet days of small magnetic range. From the mean for each column he subtracted the mean sunspot area, and expressed the difference as a percentage of this mean area. The results were as follows :—

Mean percentage difference between daily sunspot area and mean sunspot area :

on the days of largest range and the three preceding days			Selected disturbed days	and on the days of smallest range, and the three preceding days			Selected quiet days
6·8	4·9	5·1	4·4	−6·4	−5·1	−2·9	−2·5

This shows that on the average the sunspot area was slightly in excess during the four-day periods ending with the selected days of largest magnetic range, and that the excess was greater on the third preceding day than on the selected days ; during the four-day periods ending with the selected days of smallest magnetic range, the sunspot area was on the average slightly in defect, and the deficiency was greater on the third preceding day than on the quieter days. The differences were, however, slight.

In 1912 Chree applied his method more extensively to the same problem. He superposed the epochs of notable *sunspot area* (instead of magnetic disturbance), namely, the five days per month of greatest sunspot area. Moreover, he investigated both the mean sunspot area, and the mean magnetic character, of the days over 31-day intervals of time, extending *fifteen* days both *before* and *after* these epochs. The results are shown in figure 1 ; the axis of sunspot area or magnetic-disturbance intensity is vertical, and the time axis is horizontal, with the superposed selected epochs in the centre. The curve of sunspot area shows a symmetrical peak centred at the selected epoch. The curve of mean magnetic-disturbance intensity (of which in this case the chosen measure was the daily range in the horizontal magnetic intensity at Kew) is less symmetrical, but shows a notable peak four days after the sunspot peak. After examining

various subdivisions of his data in the same way, but in some cases with rather discordant results, Chree remarked that, while this result " seems to prove to demonstration that in the average year there is a close association of H ranges

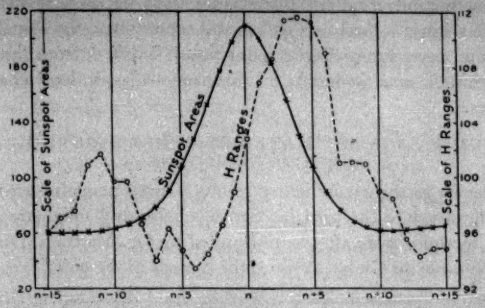


Figure 1.

with sunspot area some days previously, the relation is either of a somewhat complex character, or else is liable to be much overshadowed in individual years by other influences ".

An example of this is given by figure 2, taken from a paper written by Chree

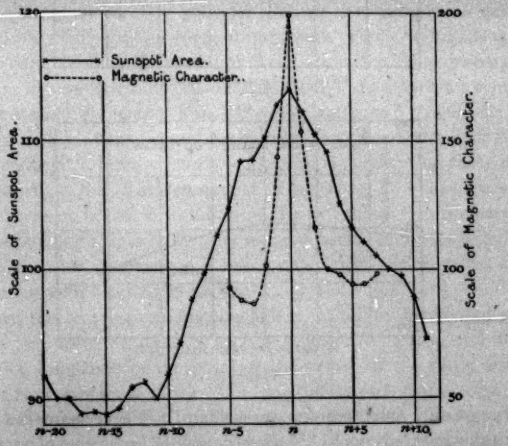


Figure 2.

in the following year, 1913. In this case the adopted measure of magnetic-disturbance intensity was the *daily magnetic character figure* (assigned by international agreement, on the basis of records from many magnetic observatories).

Figure 2 shows the mean variation of magnetic activity and sunspot area before and after superposed selected epochs, which were those of the five days of highest magnetic activity per month. The "magnetic" peak is high and narrow, indicating that notable magnetic disturbance does not persist for many days at a time. The sunspot peak is broader, and represents only a small percentage variation of the mean sunspot area. But figure 2 differs from figure 1 in that the sunspot peak is *simultaneous* with the magnetic peak, instead of preceding it by four days.

§3. TIME LAGS ESTIMATED BY CHREE'S METHOD OF SUPERPOSED EPOCHS

Perhaps owing to these conflicting results, Chree henceforward abandoned this line of enquiry into the time lag between solar and magnetic disturbance. But the same methods were afterwards applied by Dr. Maurain, Director of the Institut du Physique du Globe, Paris, to the records of the chief French magnetic

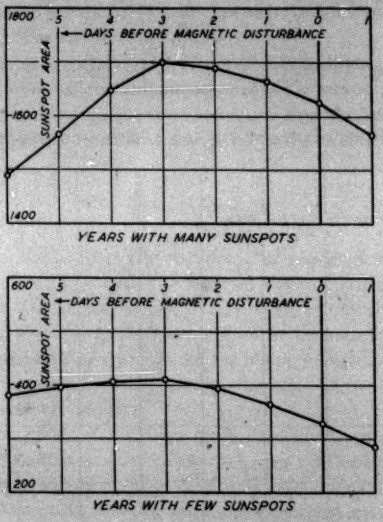


Figure 3.

observatory. Using much more material than Dr. Chree, namely 41 years' data, and taking as his selected epochs 855 days (or about 21 per year) of notable magnetic disturbance, Dr. Maurain found a shallow peak of sunspot area about $2\frac{1}{2}$ days before the magnetic peak. Dr. Stagg later found similar results (figure 3) from the Kew declination data for 1901–10, his selected epochs being days (an average of eight per year) more disturbed on the average than those considered either by Chree (60 per year) or Maurain (21 per year). The whole series of results leave

little room for doubt that, on the average, on days of notable magnetic disturbance, the sunspottedness is declining from a maximum value attained two or three days earlier. But since the sunspot peak is shallow, this relation may not show itself in every short group of years, more particularly because sunspots are at least not the sole cause of magnetic disturbance. This is shown, for example, by the occurrence of magnetic disturbance, especially in quiet years, during periods free from sunspots throughout an interval extending from five days before to three days after the magnetically active day.

§ 4. THE SUPERPOSED-EPOCH METHOD APPLIED TO THE 27-DAY RECURRENCE TENDENCY

It was in 1908 that Chree introduced his method of superposed epochs, and first applied it to determine the time lag between solar and geomagnetic disturbance ; in the same paper he applied it also to the study of an important time-relation shown in the magnetic data themselves. In order to indicate the historic setting of this work, I will go back to 1904 and describe Maunder's work on magnetic storms.

§ 5. MAUNDER'S SOLAR DIAGRAM OF MAGNETIC-STORM RECURRENCE

William Ellis, an older colleague of Maunder at Greenwich, made a list of the principal magnetic storms registered there during 22 years ; there were in all 276, or an average of 12 or 13 per year, though they were more numerous in sunspot maximum years than in sunspot minimum years. While Maunder was comparing the records of the 19 greatest of these storms with the contemporaneous state of the sun, he was struck by the fact that most of the 19 storms synchronized with the presence of a large spot near the sun's centre, and that in the few cases where this was not so, they synchronized with the return of a spot which had previously been large. Further study led him to recognize that there is a strong tendency for storms to follow one another at an interval of 27·3 days, corresponding to Carrington's mean period of rotation of the sun relative to the earth. (The sun does not rotate like a rigid body ; its rotation period increases with increasing distance from the sun's equator ; Carrington's mean period refers to a mean of the latitudes in which sunspots occur.)

To demonstrate and enforce his conclusion, Maunder proceeded as follows. He assigned a beginning and end to each of Ellis's 276 storms, and determined the heliographic longitude of the point at the centre of the sun's disc, as seen from the earth, at each of these times. The longitude of this central point, measured from a so-called prime meridian on the sun, rotating with Carrington's period, increases at a uniform rate of 360° in 27·3 days. Maunder then plotted his data on a diagram in which solar longitude was measured along the horizontal axis, and time downwards along the vertical axis. Imagine a series of horizontal lines drawn at equal vertical intervals, each line representing one rotation of the sun, occupying 27·3 days, during which the solar longitude of the central point increases from 0° to 360°. If during any such rotation one or more magnetic

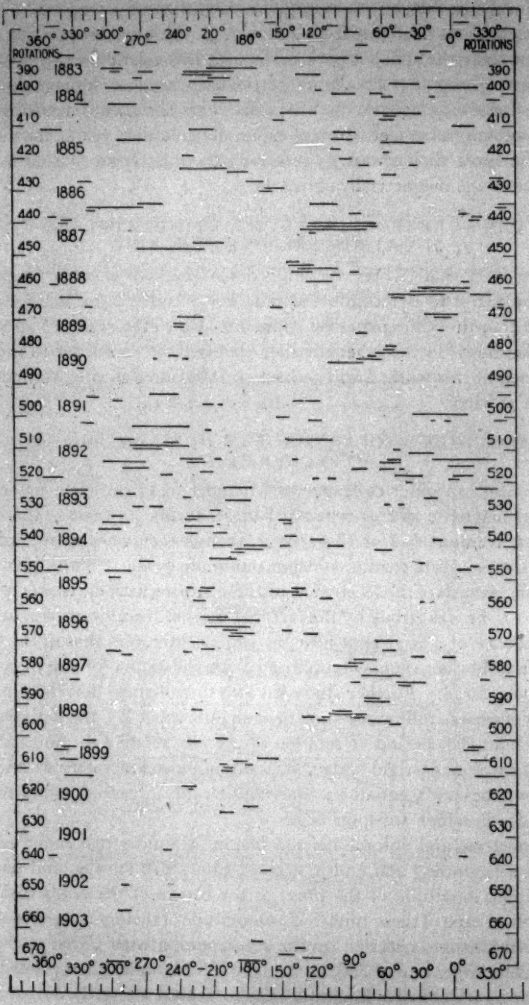


Figure 4. Maunder's diagram.

storms occurred, Maunder marked the part or parts of the horizontal line between the longitudes corresponding to the beginning and end of the storm(s). All the rest of this and other lines, not corresponding to the occurrence of a magnetic storm, was obliterated (or rather not drawn). Maunder's diagram is shown in figure 4. Its most notable feature is the frequent occurrence of lines falling nearly on the same vertical ; these represent groups of storms occurring at times when nearly the same meridian on the sun is presented towards the earth. Thus the diagram indicates the frequent recurrence of magnetic storms after an interval of one or more solar rotation periods of 27·3 days.

The tendency for magnetic storms and aurorae to recur after some such period had previously been noticed by J. Allan Broun (1858) and others ; they had called attention to a " period " of 26 or 27 or 28 days in magnetic or auroral phenomena. But its nature was not understood, and even its existence was often doubted or overlooked, until in 1904 Maunder rediscovered it and demonstrated its real character. Some earlier writers on it had thought it might be connected with the lunar month ; but Maunder, deeply imbued with a knowledge of the succession of both solar and magnetic phenomena, was quite clear as to its solar origin ; he regarded the sun as intermittently ejecting limited streams of corpuscles which, like water from a fire-hose, may sweep the earth at successive rotations of the sun, if the emission is sufficiently prolonged. His presentation of his discovery was made with this idea in view, and from the standpoint of the solar physicist.

His diagram, however, has no necessary connection with the sun ; it forms what I have called a time-pattern. The horizontal lines to be imagined in his diagram represent merely intervals of 27·3 days, and the parts of these lines actually drawn indicate merely the time interval over which the storm extended.

Maunder adopted the 27·3 interval because it is the mean solar rotation period ; but he recognized that if a succession of storms recurred at a slightly different interval, this would show itself clearly in his diagram by a series of lines in which successive members were displaced slightly to the right or left. Actually the successions of storm-lines in his diagram are rarely so clear-cut as to make it certain whether the usual interval is 27·3 days or half a day more or less than this. The identification of the mean recurrence-interval with the solar-rotation period is an independent step which, though highly probable, is a speculation rather than, like the recurrence tendency itself, an established fact.

§6. CHREE'S PULSE-DIAGRAM TO DEMONSTRATE THE RECURRENCE OF MAGNETIC ACTIVITY

The simplest method of establishing the existence of the recurrence tendency, and of determining the average length of the recurrence interval, is the superposed epoch method due to Chree. This removed every doubt as to the fact of recurrence, because the method involves no preconceived notion as to the length and significance of the recurrence interval. His proof depends only on the magnetic data, without any reference to the sun.

The selected epochs that Chree superposed were those of the five days per month that had the highest character figure of magnetic activity. The average character figures for these epochs, and for each of the five preceding and 30 or 35 following days, were calculated. The result is expressed graphically in figure 5. The average character figure naturally shows a peak corresponding to the selected epochs, but the diagram shows a repetition of the peak after 27 days; this indicates a tendency for magnetic disturbance to recur after an interval of between 27 and 28 days.

Chree applied the same method to selected epochs of the five quietest days per month; these showed the recurrence tendency equally well. In this case the

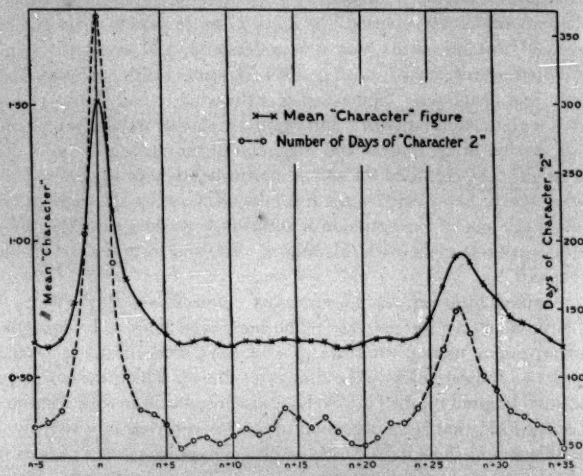


Figure 5. Superposed-epoch diagram.

initial and the repeated peak after 27 days are inverted, because their character figures are below the average.

Chree further showed that there are corresponding peaks of diminishing height (or depth) at about 54 and 81 days after the selected disturbed (or quiet) days, and also at intervals of about 27, 54 and 81 days before them; this is illustrated in figure 6, where, to save labour, the portions of the curves between the peaks were not determined.

In my opinion this beautiful and brilliant application of his superposed-epoch method of investigating time-relationships is Chree's highest achievement and best gift to geomagnetic science. It will cause his name to be remembered as long as the science is studied.

At times Chree has almost been reproached for presenting his readers

with masses of figures whose significance was not brought home to the mind by graphs ; certainly the reader's path might often with advantage have been eased in this way ; but I like to think that, nevertheless, his fame will above all be linked with his supremely successful pulse diagrams showing the recurrence tendency.

Long remembrance is not given to many, and still fewer have their name

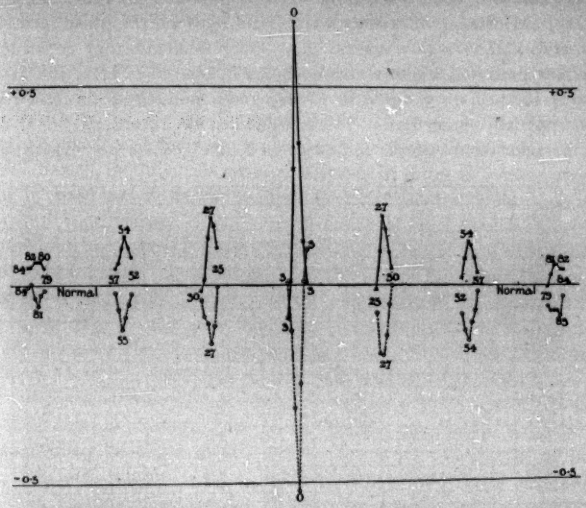


Figure 6.

associated with a visible symbol. I recall the great landmark in geometry set up when Archimedes proved that a sphere has the same area as the curved surface of the circumscribing cylinder ; it is said that he, or perhaps his disciples, had such justifiable pride in this achievement that the diagram of the sphere and cylinder was carved on his tombstone. In the same vein, I think it is fitting that the reverse of the Chree medal bears a representation of his pulse-diagram.

§ 7. THE TIME-PATTERN OF THE DAY-TO-DAY CHANGES IN MAGNETIC ACTIVITY

New ideas seldom come at once to full fruition ; the Maunder diagram of storm recurrence and the Chree pulse-diagram did not fully indicate all the available information as to the recurrence tendency in geomagnetic disturbance or its association with the sun. I will therefore carry the story further in Chree's life, and show also some of its developments since his death.

Chree investigated in great detail the significance of the recurrence pulses in his diagrams: for example, to see whether the pulse was due to a recurrence

of a few days of notable disturbance, or merely to the occurrence of more than an ordinary amount of moderate disturbance, during the period from 25 to 30 days after the selected disturbed epochs. He also examined how far the recurrence pulses and the recurrence intervals varied from year to year, and especially from sunspot maximum to sunspot minimum.

In the course of such work undertaken by him later, in 1927, in conjunction with Dr. J. M. Stagg, differences were found between the pulse recurrences in 1920–1 and 1922–3. To examine these more in detail, they constructed two 27-day time patterns, one of which is shown in figure 7. Here the days of the two years 1922–3 are ordered in 27-day rows, numbered successively. The international daily character figures (multiplied by 10) are entered on the diagram, for all the selected disturbed and quiet days (five of each per month), together

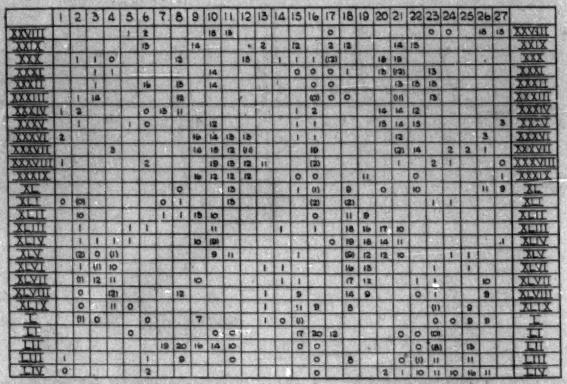


Figure 7. Distribution of quiet and disturbed days, 1922–3.

with a few additional ones (enclosed in brackets) ; detailed reasons for these additions were given.

This diagram forms a transitional stage between Maunder's recurrence-diagram and later time-patterns constructed by Professor J. Bartels. Despite its solar dress, Maunder's diagram, as I have already stated, was a true time-pattern ; each horizontal line in his diagram represented a 27·3-day period (= one rotation of the sun) ; the events indicated in his time-pattern were the periods of magnetic storms as observed at Greenwich ; they began and ended at different Greenwich times, and were of varying duration. In the Chree-Stagg time-pattern, the Greenwich day was taken as the unit of time, and the horizontal range of the diagram was exactly 27 days ; the entries were the international daily magnetic character figures, one for each selected disturbed or quiet day ; the addition of the quiet-day data extended the work of Maunder, who showed only the storm periods.

§ 8. THE BARTELS TIME-PATTERNS

The next step was taken by Professor Bartels in 1930 ; his diagram was essentially the same as the Chree-Stagg time-pattern, except that it included the character figures for *all* days. Thus it consisted simply of a rearrangement of the tables of daily character figures, which are originally published in 12 rows per year, one row per calendar month (of 28, 29, 30 or 31 days) ; Bartels reprinted the tables not according to the calendar month, but instead in successive 27-day rows. The fact that the more disturbed days have character figures involving two digits instead of one makes them recognizable at sight, and their tendency to form vertically elongated clusters covering two or more rotation-rows was evident ; similarly the eye readily recognized the location of the quietest days of character figure 0, and their similar tendency to cluster together.

In 1932 Bartels modified his table or diagram in order to present the time-pattern of magnetic activity more readily to the eye. Instead of entering, in each small square " pigeon-hole " (one for each day), the magnetic character figure for the day, he placed there a patch of black or red, partly or wholly filling the square, so as to indicate, by colour and size, the degree of magnetic disturbance (black) or quiet (red), corresponding to the magnitude of the character figure. For very quiet days, of character figure zero, the whole square was filled with red, and for the most disturbed days (figures 1·9 or 2·0) by black ; intermediate figures, from 0·1 to 0·5, were indicated by diminishing degrees of red ; a blank stood for the figures 0·6 and 0·7, and then the figures 0·8 to 1·8 were represented by increasing degrees of black. Bartels in this way constructed a diagram that showed the course of the magnetic activity for every day during more than 25 years, 1906–31 ; it represented an extraordinary concentration of knowledge, derived from magnetic observatories all over the world during a quarter of a century.

Two years later (1934) Bartels improved his diagram still further ; he abandoned the two-colour for a one-colour representation, thus not only simplifying the printing, but also indicating more appropriately, by a progressive change in the relative amounts of black and white in each daily square of the time-pattern, the gradation from extreme magnetic calm (all white) to extreme magnetic disturbance (all black). Further, he constructed a similar daily time-pattern, using the same 27-day rows, for the state of the sun, as indicated by the daily sunspot number. This seems to me to bring to perfection the time-pattern method of illustrating the day-to-day changes in the geomagnetic activity, or in the state of the sun's surface, for the separate study of each, or for their comparative study. Further improvement seems possible only by improving the data entered upon the time-pattern, and especially on the solar one.

A glance at Bartels' geomagnetic and solar time patterns for the sunspot cycle 1923–33 (figure 8) indicates that the sunspot cycle is much more strongly manifested on the sun than in geomagnetic disturbance ; the latter continues, at times with much strength, during periods when the sun is free, or almost free,

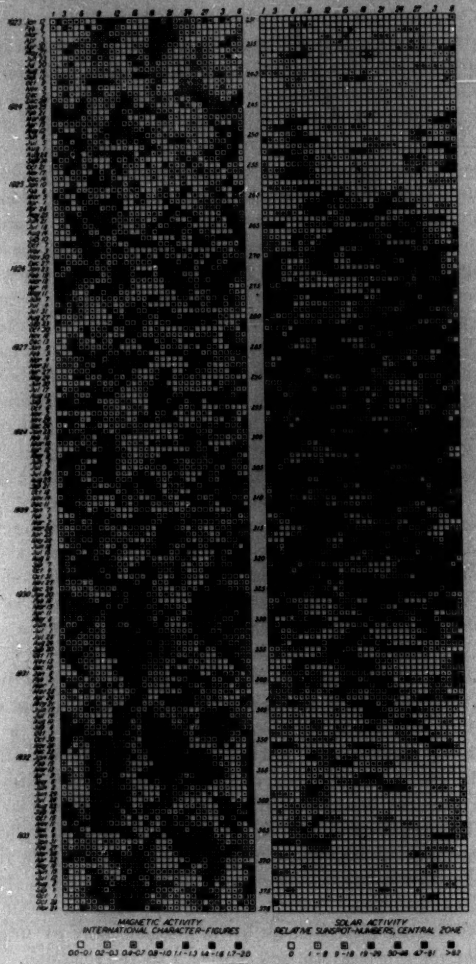

MAGNETIC ACTIVITY
INTERNATIONAL CHARACTER-FIGURES
00-01 02-03 04-07 08-10 11-13 14-16 17-20
SOLAR ACTIVITY
RELATIVE SUNSPOT-NUMBERS, CENTRAL ZONE
0 1-9 9-18 19-29 30-48 49-81 >82

Figure 8.

from spots. Even in periods when both are disturbed, their time-patterns are often very different. This makes it clear that sunspot numbers are often not a good index of the agencies on the sun that cause magnetic disturbance on the earth—a fact that Maunder and Chree emphasized. Those who study the earth's magnetism have here propounded a problem that falls properly in the domain of the solar physicist, and they eagerly await his success in attacking it.

§ 9. CHREE'S METHODS APPLIED TO COSMIC-RAY DATA

During the last few years, Chree's methods have found a new and fruitful field of application in connection with cosmic-ray data. The impact of cosmic rays upon the earth is materially influenced by the geomagnetic field. Since the measurement of cosmic-ray intensities has become precise to within a fraction of one per cent, it has been established that they undergo small time variations strongly correlated with meteorological changes. They are also correlated with geomagnetic changes.

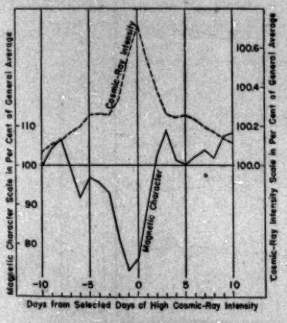


Figure 9.

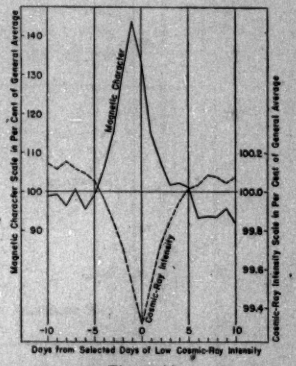


Figure 10.

Figure 9. Primary cosmic-ray intensity and magnetic character-figure pulses. For both these curves the selected " zero " days were the five days of *greatest* average *cosmic-ray intensity* in each of the 18 months, June 1938 through November 1939.

Figure 10. Primary cosmic-ray intensity and magnetic character-figure pulses. For both these curves the selected " zero " days were the five days of *least cosmic-ray intensity* in each of the 18 months, June 1938 through November 1939.

Broxon (1941) has constructed Chree pulse-diagrams, " positive " and " negative " (figures 9 and 10), showing the simultaneous average variations of cosmic-ray intensities and of the international daily magnetic character figures; the superposed epochs were those of supernormal and subnormal cosmic-ray intensity (five days per month in each case). The results should be regarded as provisional, in view of the facts related in §§ 1–3, since Broxon's data covered only 18 months; but they suggest a decided inverse correlation between the rays

and magnetic disturbance, which is supported also by other modes of analysis of the data.

The question of a 27-day recurrence tendency in cosmic-ray data has also been studied by Chree's methods.* Using Carnegie data for four stations, covering about a year at each, Gill (1939) constructed positive and negative pulse-diagrams (figure 11) of cosmic-ray intensity, which give concordant support to the conclusion that the intensity is subject to the tendency. Monk and Compton

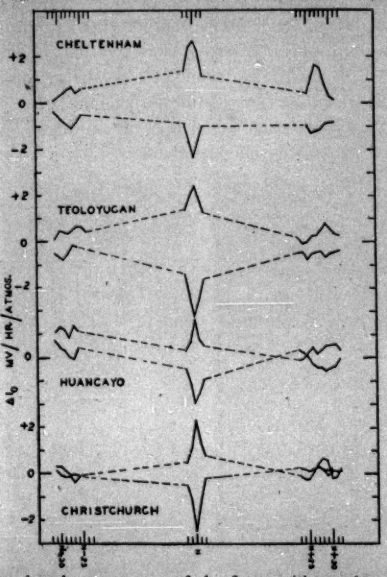
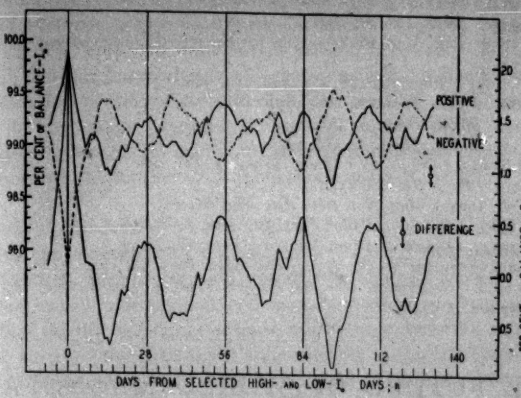


Figure 11. Curves showing the occurrence of the first positive and negative pulses after an arbitrarily chosen pulse for highest or lowest values of daily mean cosmic-ray intensity at each station.

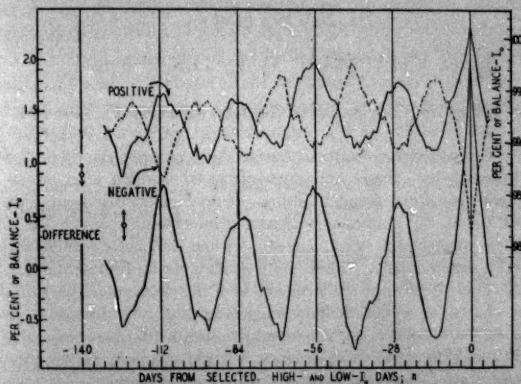
(1939) continued the investigation, using the Carnegie data (corrected for barometric changes) for one of Gill's stations, Teoloyucan in Mexico, for a longer period, nearly 2 years; unlike Gill, they determined the pulse curves not only near the pulse epochs, but also over the intervals between them. The 27-day pulses (figures 12 *a*, 12 *b*) were very marked, over several 27-day intervals before and after the selected superposed epochs. The recurrence diagrams differ from Chree's diagrams for the magnetic data in that between the pulses they show pronounced minima, instead of being nearly flat; and also, though the primary

* The same question has been studied by Forbush (1939 and 1940), using Bartels' harmonic dial methods.

pulse exceeds the recurrence pulses, the latter do not notably decrease in amplitude with increasing time-intervals from the primary epoch. These differences add



12*a*



12*b*

Figure 12*a*. Primary and following pulses (positive, negative and difference) in cosmic-ray intensity at Teoloyucan, Mexico. Zero-days of the primary pulses range from February 1937 through October 1938. Maximum probable error of any one mean is shown by the double-headed arrow.

Figure 12*b*. Primary and preceding pulses (positive, negative and difference) in cosmic-ray intensity at Teoloyucan, Mexico. Zero-days of the primary pulses range from June 1937 through February 1939. Maximum probable error of any one mean is shown by the double-headed arrow.

interest to this obviously important new field of application for Chree's methods I am convinced that the method of superposed epochs, and the study of time patterns, has much still to contribute to geophysics and cosmic physics.

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THE FUTURE OF WORLD MAGNETIC SURVEYING

BY S. CHAPMAN, M.A., D.Sc., F.R.S.

Part III of the Charles Chree Address, delivered 25 *July* 1941

ABSTRACT. The secular geomagnetic variation necessitates continual repetition of world magnetic surveying. Since iron ships displaced wooden ones, ocean magnetic surveying requires special non-magnetic ships ; it practically ceased when the *Carnegie* was lost in 1929, and the subsequent secular geomagnetic changes are imperfectly known. Magnetic surveying is slow ; one ship or one land party makes measurements at only one or two stations daily. These stations may not be representative of the area over which they chiefly determine the local course of the isomagnetic lines on the charts. In drawing the charts, some well-defined smoothing process is desirable ; continuous magnetic measurements along a world network of lines would best serve this purpose. If such records could be made on aeroplanes, a world magnetic survey might require only two or three years ; research into this possibility is desirable.

In the second part of this Address I have looked backward and reviewed some aspects of Chree's contributions to geomagnetic science, showing also how his methods are being applied in new fields.

But since the Charles Chree Medal and Prize were instituted for the encouragement of further progress in the science, as well as for the recognition of past achievements, I wish now to look forward and to discuss briefly one of the chief problems of geomagnetism, namely, the future of world magnetic-surveying.

§1. THE SECULAR VARIATION

The secular variation of the earth's magnetic field is of interest both to the natural philosopher and to the practical user of the earth's magnetism, that is to say, to everyone whose equipment includes a magnetic compass—sailors, airmen, soldiers, land and mine surveyors, and explorers. To all these it is important that our knowledge of the geomagnetic field should be maintained, and this requires continued observation. The practical user is content to know only the declination, or " variation " from true north, over the regions normally inhabited or traversed by man ; the natural philosopher wishes to know the complete world distribution of all three components of the field.

§2. EARLY MAGNETIC OBSERVATIONS AND CHARTS

During the sixteenth and seventeenth centuries, from the time of the great pioneer voyages, mariners accumulated in their logs and diaries a great store of data on the magnetic declination (and, later, of the dip), as a bye-product of their craft. Not till the end of the seventeenth century, however, was any attempt made to measure the declination systematically over a large area of the earth. This was first undertaken by Edmond Halley. His primary motive was purely scientific, though he had a proper regard for the needs of practical men. He made two voyages (1698–1700) in the North and South Atlantic Oceans, in command of a naval vessel placed at his disposal by King William III; doubtless he himself made many, perhaps all, of the magnetic observations.

On his return he devised a new method of representing his results on a magnetic chart, thus giving most valuable aid to seamen; his method is still used to-day, substantially unaltered. After first publishing (1701) an Atlantic chart based on his own observations, he collected those made by other sailors, and extended his magnetic lines over the Indian and Eastern Pacific Oceans, on a World Chart (1702).

In 1768 Wilcke published the first world chart of magnetic dip or inclination; one stimulus to the observation of this magnetic element was the hope that it would assist mariners in determining their longitude.

The measurement of the magnetic intensity in mechanical units was first made possible by Gauss, in 1832. Comparative measurements had been made earlier in the nineteenth century, and early magnetic charts for the intensity were constructed, for areas of increasing extent, by Hansteen (1825), Duperrey (1833) and Sabine (1837).

§3. LATER MAGNETIC SURVEYS AND CHARTS

During the nineteenth century, the change from wooden sailing ships to iron-built steamships made it more and more difficult for sea captains to aid magnetic science, as they had hitherto done, by making magnetic measurements on their voyages. Our geomagnetic knowledge came to depend, over the sea as over the land, on specially organized expeditions, in which Great Britain

took a notable share, as befits the chief maritime nation. The Admiralty undertook the preparation of the world magnetic charts, and its survey vessels and other British ships made valuable magnetic measurements over the oceans. Other nations also shared in these endeavours and in the preparation of their own magnetic charts.

Never, however, has the world magnetic survey been adequate from the standpoint of pure science; the inner changes of the earth set a pace with which the observers have hitherto been unable to keep up. The changes of the earth's magnetic field are swift for a phenomenon on a scale so great, and the localities of most rapid change shift unpredictably from place to place.

§4. OCEAN MAGNETIC SURVEYS BY NON-MAGNETIC SHIPS

At the beginning of the present century, Louis Bauer saw the need to improve the provision for the world magnetic survey, and he was able to gain the ear of Andrew Carnegie, whose wise and generous benefactions included the foundation of the Carnegie Institution of Washington. A Department of Terrestrial Magnetism was formed (1904) as part of this Institution, and one of its first activities was the organization of magnetic observation at sea on a special sailing vessel, the *Galilee* (1905–8): later a non-magnetic yacht, the *Carnegie*, was built, which did fine service during the years 1909–21. The department also made or co-operated in several land magnetic surveys. The Admiralty magnetic charts (for all three elements) for 1922, produced soon after the conclusion of the *Carnegie's* first six voyages, were largely based on the Department's data, published with praiseworthy promptitude. These charts represent the high-water mark in man's knowledge of the geomagnetic field at any epoch. Yet, great as was that achievement, to which many nations added their contribution, it fell far short of the ideal, both in accuracy and completeness.

Since then we have been losing ground; the magnetic field has continued to change, without our having adequately followed its changes. In 1928 the yacht *Carnegie* resumed her survey work, after having been laid up for seven years; her destruction by fire in 1929, after completing only half her projected three-year cruise, was truly a disaster for geomagnetic science. The Carnegie Department found itself unable to replace her, and for some years the outlook for a resumption of world magnetic surveying remained dark. Then, not long before the present war, the British Admiralty undertook the building of a new non-magnetic vessel, the *Research*, under the control of the Hydrographer. But for the war this ship should by now have been well advanced on her first survey cruise.

Even while hopes were still bright concerning the work of the new vessel, the *Research*, the American Geophysical Union was urging the United States Government to follow the British example, and build another non-magnetic survey vessel. This was in order that the United States Coast and Geodetic Survey might co-operate by extending to wider areas the fine service it has for

many years rendered to magnetic science, by magnetic surveys on American territory and in the neighbouring waters.

Clearly the renewal of world magnetic surveying on any adequate scale must await a happier future; yet even in these dark days one may consider how best this task can be most effectively and economically performed.

§ 5. THE TIME REQUIRED BY PAST MAGNETIC SURVEYS

On the last completed cruise of the yacht *Carnegie*, in two years (1919 Dec. 9 to 1921 Nov. 11) she was at sea for 487 days, and the distance traversed was 64 000 miles; thus the average daily travel was 132 (nautical) miles. On the average, the magnetic declination was observed almost twice daily, at sea, at intervals of 77 miles; the other two magnetic elements were measured not quite once daily, at an average interval of 146 miles.

As the oceans cover the greater part of the earth, they are of most importance in a world magnetic survey. But of course the land areas cannot be neglected. Let us therefore consider the most recent complete magnetic survey of the British Isles, which was made in the summers of 1914 and 1915 by G. W. Walker, repeating earlier surveys by Rücker and Thorpe. At each of the 183 stations occupied, all three elements were measured. The average distance between the stations was 50 miles or less, and rarely was more than one station occupied daily; the average over the observing period was more nearly one in two days. The observers were fortunate in being able to travel by motor car over good roads, and particularly in having the co-operation of the Ordnance Survey. The officers of the Ordnance Survey chose stations suitably undisturbed by local causes, natural or artificial; they located the stations exactly; and they provided an accurate azimuth at each station. Observers at sea are without such aids. Clearly magnetic surveying is a slow business, whether on land or sea. Naturally, owing to the motion of the vessel and other causes, sea observations aim at a lower standard of accuracy than land observations, and the chief artificial local magnetic disturbance to be feared is that of the ship itself.

§ 6. MAGNETIC CHARTS BASED ON ABSOLUTE OBSERVATIONS

In representing the survey results on charts, by means of isomagnetic lines, a difficulty arises, especially on land; the difficulty is less the more ignorant we are, or the less complete is our survey. When the observing points are widely spaced on the map, smooth isomagnetic lines consistent with them can usually be drawn; but generally this is the less possible the closer the network of points, if one has faithful regard to the data. It is necessary to make the isomagnetic curves twist and turn, and inevitably the query arises whether we would arrive at the same form of the lines if the network of observing points, though of equal number, had been differently located.

In any case, though a complicated series of lines may be shown on large-scale maps of Great Britain, they cannot properly be included without incongruity

on a world magnetic chart. On such a chart there should be a certain uniformity; the lines should not show relatively minute local peculiarities known to exist over a few specially well-surveyed areas, while omitting them, perforce, over the major area where they are not known. The representation must necessarily be a smoothed one. But no clear principles of smoothing, and of representing the departures from the smoothed field, have yet been generally agreed upon. As regards the ocean observations, it is uncertain to what degree they are liable to natural local disturbance, though over the deep seas, far from continents and islands, this is likely to be less than in land areas.

If one had complete information about the geomagnetic field, the most reasonable procedure would seem to be to divide the earth's surface into an appropriate number of areas regularly distributed and of nearly equal extent, and to base the magnetic charts on the average value of each component of the field over each such area. Charts so constructed would give a properly smoothed version of the field, and could be supplemented by auxiliary charts indicating in some way the local departures from the smoothed field, or at least the distribution of the range of the local departures.

This is a counsel of perfection, to which one might approach if one could determine the three elements of the earth's field at a close regular network of points covering the globe. The statistics of local disturbance would necessarily depend on the length of the intervals between these points; the only obvious way of determining exactly the average field components over an area would be one based on flux measurements through coils bounding the area, or on induction in long wires transported across the area. This, of course, is impracticable for large areas, but it may serve to point to a better method than the present system of absolute observations at isolated stations. Probably the best approximation we can hope to make to an average over an area is an average based on observations made continuously in traversing one or more appropriately situated lines crossing the area.

§7. MAGNETIC SURVEYS BY TRANSPORTED CONTINUOUS RECORDERS

This plan involves the abandonment, as the main method of the survey, of absolute observations at large numbers of stations. The partial replacement of absolute observations by comparative observations has already been advocated by G. W. Walker, and is a common practice in mineral or oil prospecting by magnetic methods. My own proposal involves abandonment not only of the absolute measurements (except at key points), but also of the individual stations.

Observations at isolated stations necessarily involve serious difficulties. The chief object of the continued repetition of magnetic surveys is to determine the rate of secular change of the earth's field; the standard method is to re-occupy the same site in successive surveys. But the erection of nearby buildings, or new electric-traction undertakings within a considerable distance, may render a

site unsuitable for re-observation. Also, as Dr. J. A. Fleming (1939) has remarked, " often when a place has been visited after an interval of years it is impossible to find and make a re-determination at the same position as that previously used, and a new point must be chosen. But as the time-changes sought are relatively small, and as local displacements often cause considerable differences, due solely to peculiarities of the local geology, a new uncertainty is introduced to increase the difficulty of the task ".

This uncertainty would obviously be lessened if our charts and the determinations of secular variation could be based on magnetic measurements made by *continuously recording* instruments transported along lines crossing the network of areas chosen for averaging.

This conclusion can be of importance only if such continuous measurements with instruments in transport can be made with sufficient accuracy. Such a thing has, to my knowledge, never been attempted, and there are obvious difficulties. The accuracy aimed at in geomagnetic measurements is high, and the *absolute* instruments usually employed are delicate; they can be transported, but are not designed for continuous observation, and their directions must be known with great precision. The existing *continuous* magnetic recorders, on the other hand, are designed to work at a fixed point. Clearly new instruments would have to be devised and perfected by experience. Hence I should certainly not advocate an early abandonment of present methods of magnetic surveying; the next world magnetic survey should probably be made, or at least begun, on the old lines, and the non-magnetic vessel, the *Research*, has, I hope, much to contribute to geomagnetism. But while this is proceeding there is need, I believe, to develop new methods of observation.

The instruments employed must be robust, and as it would probably be impracticable to keep their orientations constant, their changes of orientation must be recorded *; this would increase the difficulty of interpreting or, as the astronomers say, " reducing " the observations, but such work can be done in an office, at leisure, and with every useful auxiliary device of measurement and computing from the records.

According to the proposed plan, the data should be provided in the form of continuous records of the magnetic elements by instruments in transport. The continuous registration of the magnetic changes at fixed observatories is an established process, but new problems of design and interpretation would arise in connection with instruments registering while in transport from place to place; electromagnetic instruments would probably be preferable to purely magnetic ones. The records would include variations due to lapse of time as well as change of place, and the time variations would need to be allowed for on the basis of knowledge provided by fixed observatories. For this purpose

* Problems of this kind arose in connection with the 1935 stratosphere flight by members of the U.S. Army Corps in the balloon ' Explorer II ' ; see p. 139 of National Geographic Society, Contributed Technical Papers, Stratospheric Series, no. 2, 1936.

the network of magnetic observatories would have to be moderately increased. As part of the calibration of the continuous records, absolute observations might be made on land at the beginning and end of each flight.

§8. MAGNETIC SURVEYING BY AEROPLANE

If suitable instruments could be devised, their transport could probably best be made by aeroplane. This would have a great advantage in speed, as compared with transport by ship or motor car; and large aeroplanes, which can travel at heights well above the turbulence and the clouds of the troposphere, would probably be steadier, on the average, than either ships or land transport. The disturbing influence of iron buildings and electric traction on land would be removed or greatly lessened. It is probable that the disturbing influence of the aeroplane itself can be adequately reduced throughout an interior space sufficient to accommodate the magnetic recording instruments, by adaptations no greater than those required in connection with non-magnetic survey ships.

I am not competent to compare the capital cost of a partly non-magnetic aeroplane, and its weekly running costs for operating and scientific personnel, ground staff and fuel, with those for a non-magnetic ship, but the results achieved, say per week, should be far greater with the aeroplane than with either the ship or a land survey party. If the weekly distance travelled were 10 000 miles, the records would cover routes that would take ten times as long to traverse by ship, without any allowance for days in port. Moreover, provided that the records could be interpreted to give the magnetic elements over a time-interval of, say, a quarter of an hour, with as great an accuracy as that of a single complete determination of the elements on board ship, the smoothing provided by the aeroplane's moving record would be advantageous. The interpretation would certainly be more costly and troublesome, because the computers at the central office would have to deal not only with the magnetic records, but also with the records of orientation of the aeroplane and its instruments, and with the navigational records needed to time the height and geographical course of the aeroplane. Finally, the observations would have to be reduced to ground level, a task, however, which should not offer undue difficulty. The geographical position of the aeroplane would need to be known within a few miles in order to determine its *mean* position over any quarter-hour with sufficient accuracy; if the aeroplane travelled above the clouds, celestial objects for the determination of position would always be available.

As the aeroplane would spend several hours, at least, on land, before each flight, the staff could assure themselves, by suitable magnetic observations at their starting point, or by a radio message from a magnetic observatory, that the day was appropriate for magnetic observation; if a magnetic storm were in progress the flight could be postponed. In magnetic surveys by ships, a magnetic storm may render one or two days' observations useless, and so cause a gap in the survey.

Within two or three years the whole earth could be covered by an adequate network of flights, over the poles and the great oceans as well as over difficult land areas; one might thus hope to achieve a far more complete world magnetic survey than ever before, substantially at one epoch. Afterwards a programme of repetition flights over a more open network would reveal the areas of most rapid secular change, and these could be surveyed in more detail, and at more frequent intervals, between the repetitions of the whole survey.

Naturally numerous difficult technical problems are involved in such an enterprise, demanding the co-operation of workers of many kinds: to name but a few, the aeroplane designer must determine whether a suitable space in the aeroplane could be rendered sufficiently non-magnetic; the magnetic instrument designer must consider whether suitable recording instruments can be devised; the aerial navigator must advise regarding the steadiness of the aeroplane, and the accuracy of the determination of its height and course; and the enterprise would certainly make great calls on the organizers of air transport.

I hope that in due time these proposals, here tentatively made, will be considered and discussed by persons with the necessary detailed knowledge. I believe that there is at least a *prima facie* case for their consideration, and, if a closer preliminary examination is favourable to them, for the expenditure of much money on preparatory researches of various kinds. The goal is a great one—a reasonably accurate and complete world magnetic survey in a reasonable time, with a hope of great ultimate saving of cost as well as of better, because properly smoothed, results. Finally, I think there is little doubt that other objects of both practical and scientific importance could be achieved by making additional types of observation, notably meteorological, on the same series of scientific flights. Hence, though I realize that my picture of future world magnetic surveying is merely a dream, and an easy dream, I hope that time will show it to be not altogether an empty dream.

REFERENCES

For further information concerning magnetic surveying (its history, results, present methods and instruments, and bibliography) the reader is referred to Chapters I and II of the book edited by J. A. FLEMING (see below), and to Chapters I–IV and XXVI of *Geomagnetism*, by S. CHAPMAN and J. BARTELS. The following are references to work specially mentioned in §§ 2, 4, 5:—

Carnegie Institution of Washington, 1912–27. *Researches of the Department of Terrestrial Magnetism*, Publication no. 175, vols. I–VI.

CHAPMAN, S., 1941. *Occasional Notes R. Astr. Soc.* 1941 ("Edmond Halley and his Charts").

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THE THERMAL EXPANSION OF LEAD FROM 0° C. TO 320° C.

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Received 26 *July* 1941

ABSTRACT. The variation with temperature of the lattice parameter of lead has been measured, and the coefficient of expansion deduced. The results are expressed by the equations :

$$d = 4{\cdot}93687 + 138{\cdot}98 \times 10^{-6} \,.\, t + 60{\cdot}16 \times 10^{-9} \,.\, t^2 \text{ A.},$$
$$\alpha = 28{\cdot}15 \times 10^{-6} + 23{\cdot}6 \times 10^{-9} \,.\, t.$$

The error in d is about 0·0001 A., and in α, $0{\cdot}3 \times 10^{-6}$.

§ 1. INTRODUCTION

The work described in this paper represents further development of the use of the high-temperature camera designed and constructed in this department (Wilson, 1941). As a prelude to its use for the investigation of the structure of magnetic materials at high temperatures, it was considered advisable to explore fully its possibilities as a precision instrument at lower temperatures. The thermal expansion of lead, which does not seem to have been measured previously by means of x rays, was taken as a subject for this purpose.

§ 2. APPARATUS

The method used has been fully described by Wilson (1941). The lattice parameter is calculated from measurements of Debye-Scherrer photographs taken with the specimen maintained at the required temperature. The only change in the apparatus has been the addition of an automatic temperature control. The furnaces of the camera were made one arm of a Wheatstone bridge, and the bridge was balanced with the furnaces at the correct temperature. Any change in temperature put the bridge out of balance, and thereby deflected the galvanometer needle of a thermostatic controller. This switched a series resistance in or out of the circuit as required (White and Adams, 1919). The arrangement was reasonably satisfactory, keeping the temperature within 1° C. of the required value without much attention.

§ 3. MEASUREMENT OF TEMPERATURE

The temperature of the specimen was measured by means of a platinum *vs.* platinum-rhodium thermocouple and a potentiometer reading to one microvolt. The thermocouple was calibrated as follows :—

(1) At room temperature, by comparison with a mercury thermometer.

(2) At the steam point, under measured barometric pressure.

(3) At the melting points of urea (132°·7 C.), tin (231°·85 C.) and lead (327°·3 C.). Specimens of these were put in thin glass or silica tubes and suspended in the positions normally occupied by the specimen, and were observed through a small telescope. Potentiometer readings were taken at the temperatures at which the specimens were found to melt and to solidify. The urea could be seen to melt or recrystallize; in the cases of lead and tin the state of the specimen was observed by pushing a quartz rod into it.

(4) At the melting point of lead (327°·3 C.). The temperature of the specimen was maintained (by hand, as the automatic control was not quite accurate enough) at some temperature near the melting point while a photograph was taken. The presence of lead lines on this photograph would show that the lead was solid: their absence would show that it was molten. By taking photographs at suitable temperatures, the limits between which the melting point must lie were narrowed down until they were about $\frac{1}{3}$° C. apart. The melting point of lead by the two methods agreed within 3 microvolts (0°·3 C.).

These melting points were compared with the standard values (Adams, 1914) and a graph was plotted showing the corrections to be added to the potentiometer readings. The largest correction was about 20 μv. or 2° C. It should be noticed that in the cases (3) and (4) the thermocouple and specimen were arranged exactly as for taking a photograph, and therefore the corrected temperature should give that of the specimen, even though the specimen and thermocouple may not be at the same temperature. This is not so in cases (1) and (2), but here we are dealing with low temperatures, at which we should not expect much discrepancy.

§ 4. ANALYSIS OF THE LEAD

The analysis of the lead used was given by the makers as: Pb 99·997 %, Ag 0·0006 %, Cu 0·0009 %, Sb 0·0007 %, Bi 0·0002 %. It was part of a sample whose heat capacity had been measured by Bronson and Wilson (1936).

§ 5. RESULTS

Measurements from 17 photographs were used in the final results. They were taken at roughly 50° intervals from room temperature to the melting point, two being taken at most temperatures. The measurements are given in the table on p. 660. A graph of lattice parameter against temperature showed that

the results could probably be expressed by a relation involving t and t^2, and the best-fitting relation of this form was found by the method of least squares to be

$$d = 4{\cdot}93687 + 138{\cdot}98 \times 10^{-6} \,.\, t + 60{\cdot}16 \times 10^{-9} \,.\, t^2 \text{ A.} \qquad \ldots\ldots(1)$$

The values of d calculated from this equation, and their deviations from the observed values, are also given in the table. As there are no systematic deviations, equation (1) is taken as representing the results satisfactorily. Although in principle it is not very satisfactory to limit oneself in this way to a power series ending in t^2, this is probably as good a representation as can be obtained.

To find the coefficient of expansion, we calculate from equation (1) the value of $\frac{1}{d} \cdot \frac{\partial d}{\partial t}$ as a linear function of t, giving

$$\alpha = 28{\cdot}15 \times 10^{-6} + 23{\cdot}6 \times 10^{-9} \,.\, t. \qquad \ldots\ldots(2)$$

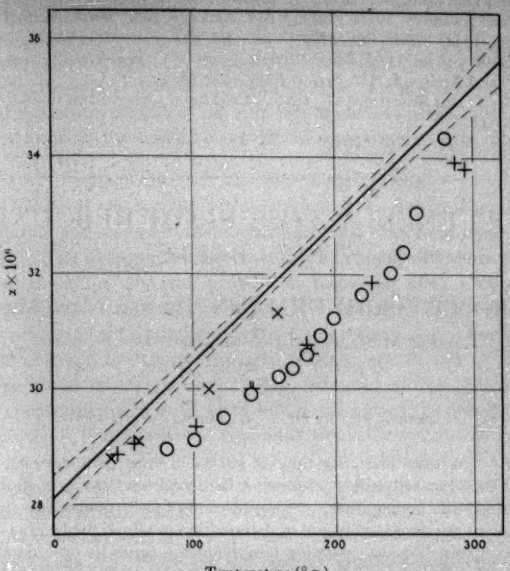
Table

t (° c.)	$d_{\text{obs.}}$	$d_{\text{calc.}}$	Diff.
12·6	$4{\cdot}9387_2$	4·93863	$-0{\cdot}9 \times 10^{-4}$
14·5	$4{\cdot}9387_6$	4·93889	+1·3
14·6	$4{\cdot}9388_0$	4·93891	+0·2
18·6	$4{\cdot}9393_6$	4·93947	+1·1
58·2	$4{\cdot}9450_8$	4·94516	+0·8
60·1	$4{\cdot}9455_5$	4·94544	−1·1
97·9	$4{\cdot}9511_1$	4·95105	−0·6
99·3	$4{\cdot}9514_2$	4·95126	−1·6
150·6	$4{\cdot}9591_1$	4·95916	+0·5
158·4	$4{\cdot}9603_2$	4·96039	+0·7
196·7	$4{\cdot}9665_6$	4·96653	−0·3
208·2	$4{\cdot}9683_9$	4·96841	+0·2
249·8	$4{\cdot}9755_2$	4·97534	−1·8
254·8	$4{\cdot}9762_0$	4·97619	−0·1
300·6	$4{\cdot}9838_4$	4·98408	+2·4
302·3	$4{\cdot}9843_5$	4·98438	+0·3
320·8	$4{\cdot}9877_7$	4·98764	−1·3

The total error in the individual observed lattice parameter is probably not more than 0·0002 A. The errors in the calculated values of d and α have been derived by a method given by Whittaker and Robinson (1926). These errors vary with temperature, being smallest at the centre of the temperature range, and increasing toward the extremes. The largest calculated error in the value of d is 0·0001 A., and that in α is $0{\cdot}3 \times 10^{-6}$.

The equation for α is plotted in the figure, with the results of Uffelmann (1930), Hidnert and Sweeney (1932) and Eucken and Dannöhl (1934) for comparison. The limits of error are shown by the dotted line on either side of the full line representing the expansion coefficient. The present results are seen to be somewhat higher than previous measurements, which were all made by macroscopic

methods. It is difficult to see how the difference in method could account for this, as one would expect the lattice parameter to vary in the same way as the dimensions of the lead *en masse*.



The expansion coefficient of lead.
Full line, authors' results ; ⊙ Uffelmann ; × Hidnert and Sweeney (mean coefficient plotted against mean temperature) ; + Eucken and Dannöhl.

The results do not confirm Grüneisen's law. The coefficient of expansion increases about two and one-half times as fast as the heat capacity, as is shown in the following comparison:

	0°	100°	200°	300° C.
$\alpha(t)/\alpha(0)$	1·000	1·084	1·168	1·252
$C_p(t)/C_p(0)$.....	1·000	1·038	1·064	1·096

The C_p ratio at 100° is taken from the work of Bronson and Wilson (1936); those at 200° and 300° from the summary by Kelley (1934).

ACKNOWLEDGEMENTS

The authors desire to express their thanks to Professor Sir Lawrence Bragg, F.R.S., and to Dr. A. J. Bradley, F.R.S., for their encouragement and interest ; and to Trinity College (A. R. S.) and the British Electrical and Allied Industries Research Association (A. J. C. W.) for financial assistance.

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RATE OF ENERGY LOSS FROM REBOUNDING STEEL BALLS

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Communicated by Dr. P. B. Moon, 7 April 1941

ABSTRACT. The total bouncing time of hardened steel balls released from given heights above the practically plane surfaces of hardened steel anvils of limited size has been measured at room temperature. The coefficient of restitution is very little different from unity in the sensibly elastic region of deformation in which the heights of fall of the sphere plotted against the corresponding total bouncing times lie on a parabola. The multiple rebounds sufficiently integrate the small energy loss, so that the well reproducible total bouncing time varies measurably when the coefficient of restitution is altered by an exceedingly small amount.

For a given material and initial height of fall, the smaller sphere exhibits the longer bouncing time. Defective spheres show striking deviations from this rule. The total bouncing time increases with the height of the anvil until this has reached a certain value.

Lunette-shaped markings have been observed on the highly polished surface of the anvils as the result of a great number of rebounds.

§1. INTRODUCTION

THE significance of establishing a temperature coefficient of solid friction (between naked metal surfaces) as a criterion to distinguish between different mechanisms postulated for this effect has been indicated elsewhere (Schnurmann, 1941). Independently of this, an experiment which involves deformation only should be done over a wide range of temperatures to determine any difference between the order of magnitude of the temperature coefficient in the case of a friction experiment and that of an experiment in which no action other than elastic deformation is involved Over a wide range of temperatures,

from the melting point of a material to absolute zero, the rate of energy dissipation by elastic deformation should depend upon the temperature of the system, since the adiabatic compression energy should tend to vanish upon approaching the absolute zero of temperature, and also because the thermal conductivity depends upon temperature. The rate of energy dissipation when a solid sphere rebounds from the surface of a finite mass of elastic solid has been studied at room temperature, and apparatus has been designed for making similar measurements at various temperatures of the sphere and anvil *in vacuo*, but owing to the present circumstances the manufacture of this apparatus has been suspended.

§ 2. THE TOTAL BOUNCING TIME

The coefficient of restitution* e is a measure of the energy dissipation during a single rebound, and assumes values which are very close to unity in the sensibly elastic range. The accurate measurement of the small difference between the initial potential energy and the proportion recovered after a single rebound of the sphere would be rather difficult, and it is more profitable to use a method of multiple rebounds which integrates the small energy loss. For this purpose a sphere was released from a given height H above the surface of a finite anvil, and the time (total bouncing time t) was measured during which the sphere performed consecutive rebounds until it had dissipated the whole of its initial potential energy. The number of rebounds would be infinite, of course, if the coefficient of restitution were constant throughout the decrease of the amplitude of the oscillating sphere, provided that the sphere did not come to rest when its potential energy decreased below a certain small value mgH_f (m = mass of the sphere; H_f = height of fall for which $e=0$). In this ideal case the (H, t) curves would be parabolas, i.e. the ratio H/t^2 would assume a constant value. The result of a summation of the time of oscillations is that the nth impact takes place when the time t_n has elapsed, where

$$t_n=\left(\frac{2H}{g}\right)^{1/2}(1+2e^{1/2}+2e+2e^{3/2}+\ldots+2e^{(n-1)/2}).$$

Since $0<e<1$ for $n\to\infty$, the total bouncing time is

$$t=\left(\frac{2H}{g}\right)^{1/2}\frac{1+e^{1/2}}{1-e^{1/2}},$$

so that $t=\infty$ for $e=1$

and $t=(2H/g)^{1/2}$ for $e=0$,

while
$$\frac{H}{t^2}=\frac{g}{2}\left(\frac{1-e^{1/2}}{1+e^{1/2}}\right)^2 \qquad \ldots\ldots(1)$$

is constant for a given value of e.

In the physical case, the number of consecutive rebounds must be finite, though large, for any measurable value of the height of fall, and the experiments

* The coefficient of restitution is defined here as the ratio of the total mechanical energies of the sphere after and before the impact.

described below show that conditions can be established for which the values of H/t^2 are practically constant over an appreciable range of values of the initial height of fall of the sphere. The accuracy with which the time t can be measured depends upon the accuracy with which the end point of rebounding can be observed. This end point is well defined because in its neighbourhood the sphere appears steadily to approach its mirror image in the polished surface of the anvil, since the frequency of the oscillating sphere becomes too large for the eye of the observer to follow its individual oscillations.*

§ 3. EXPERIMENTAL PROCEDURE

The sphere was released from a recessed electromagnetic holder, the centre line of which was set up to coincide with that of the anvil. The highly polished surface of the cylindrical anvil was slightly concave (mean depression in centre 0·1 cm.) and tended to bring the rebounding sphere back to vertical motion in case of an external disturbance, which otherwise would cause the sphere to leave a flat anvil after a few rebounds.

It is essential to record always the *maximum* value of the total bouncing time for any given initial height of fall and temperature, because the possible errors (such as an external disturbance, spin of the sphere, contamination of the contacting surfaces) all tend to diminish the value of t. The deviation from linear oscillations of the sphere indicates the occurrence of an external disturbance. A spin which the sphere may have acquired produces a rolling motion which is superimposed on the oscillations of the sphere, particularly when the amplitudes become small. Finally, in this case the rebounds die out before the rolling motion. The mechanical cleanliness of the contacting surfaces influences the value of t, because particles of dust and liquid films are deformed during the impact and may even be lifted up from the surface of the anvil. In such circumstances the sphere has been observed to fall abruptly to rest from a small height above the surface of the anvil instead of steadily approaching the end point of rebounding. In the experiments in air described below, mechanical cleanliness was maintained by wiping the surfaces of the sphere and the anvil with a clean chamois leather before each release of the sphere from the magnetic holder.

The part played by the air resistance is very small in these experiments in the region where the H/t^2-values are approximately constant, so that the ratio of the energies lost in one complete cycle of oscillation $E_r/mgh(1-e)$ is small (E_r = energy loss on account of the air resistance). Four sizes of steel balls were used, the largest having a diameter $d=1{\cdot}270$ cm. and the smallest $d=0{\cdot}476$ cm. $E_r=2\int_0^h (C_D \times \frac{1}{2}\rho v^2 \times \frac{\pi}{4}d^2)\,dx$ for one complete cycle of oscillation of the sphere, so that

$$\frac{E_r}{mgh(1-e)} = \frac{\pi}{4} C_D \frac{d^2\rho}{m} \frac{h}{1-e}, \qquad \ldots\ldots(2)$$

* The individual rebounds are audible when the experiments are made in air, even down to the end point of rebounding.

C_D being the drag coefficient and ρ the density of air. Anticipating the result recorded below that $1-e=0{\cdot}010$ for $d=0{\cdot}476$ cm. when $h\leqslant 11$ cm., and assuming $C_D=0{\cdot}5$, one obtains from equation (2) $\frac{E_r}{mgh_1(1-e)}=0{\cdot}288$ for $h_1=11$ cm., and $\frac{E_r}{mgh_2(1-e)}=0{\cdot}046$ for $h_2=1{\cdot}75$ cm.

Four different anvils of the same diameter (see table) were used. Two of them, T_1 and T_2, were made of the same bar of a tool steel (0·75 to 0·85 % C, 0·35 % max. Mn) and varied in height. The other two anvils, S_1 and S_2, were made of a Skefko ball-bearing steel (1 % Cr), one of them having the same dimensions as the larger anvil made of tool steel.

Anvil	Material	Diameter (cm.)	Height (cm.)
T_1	Tool steel	6·74	1·9
T_2	Tool steel	6·74	6·8
S_1*	Skefko ball-bearing steel	6·74	6·8
S_2	Skefko ball-bearing steel	6·74	15·1

§ 4. RESULTS

The total bouncing time was always found to be *larger* for the *smaller* sphere at a given initial height of fall and with a given anvil (figure 1). The anvil S_1 proved more suitable for observing approximately constant H/t^2-values over an appreciable range of heights of fall than the anvils T_1 and T_2, particularly when the smallest sphere (0·476 cm. diameter) was used. In this case the H/t^2-values were approximately constant up to an initial height of fall of $H=11$ cm. (figure 1); thus $H/t^2=0{\cdot}0026$ for $H=1{\cdot}75$ cm. and $H/t^2=0{\cdot}0030$ for $H=11$ cm. A value of $e=0{\cdot}990$ was computed for $H/t^2=0{\cdot}0028$. For $H>11$ cm., the ratio H/t^2 showed a clear tendency to rise (for instance, $H/t^2=0{\cdot}0050$ for $H=16{\cdot}25$ cm. when $e=0{\cdot}98_8$). The results obtained when a steel ball of 0·794 cm. diameter rebounded from anvil S_2 were almost identical with those (figure 1, curve *b*) obtained with anvil S_1, and proved that an increase of the height of the Skefko anvil S_1 would not lead to larger total bouncing times.

The curves drawn in the (H, t) diagrams (see figures) form the envelopes of all the measured maximum values of the total bouncing times. With anvils T_1 and S_1 the largest deviation of measured maximum values of t from the curves drawn is 10·5 % and 3·4 % respectively. There is no indication that these few deviations, which are small in the case of the Skefko anvil, are due to anything more than observational errors. More pronounced deviations were observed with the long tool-steel anvil T_2. They have the appearance of a hump which was observed with this anvil for

* The author is indebted to Mr. H. Zetterström (Messrs. Skefko Ltd., Luton) for the supply of this anvil.

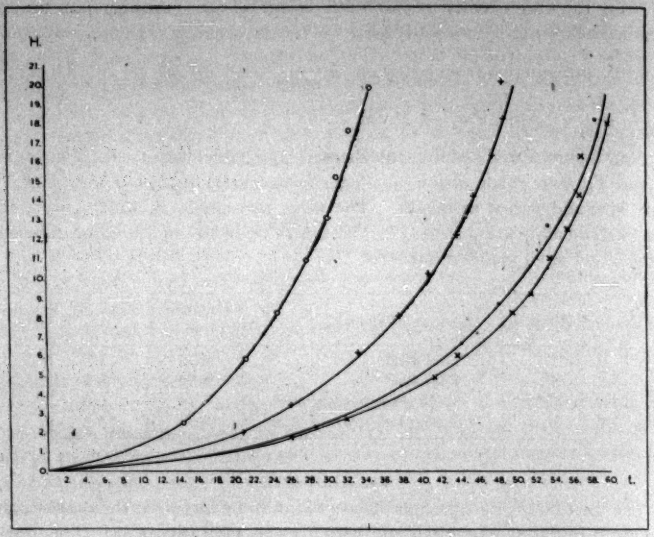

Figure 1, curve *a*. Maximum values of total bouncing times (t sec.) of Skefko steel balls on Skefko anvil S_1 ($H_{D50}=875$) from heights H cm.

× ball diameter 0·476 cm. + ball diameter 1·000 cm.
● ball diameter 0·794 cm. o 1·270 cm.

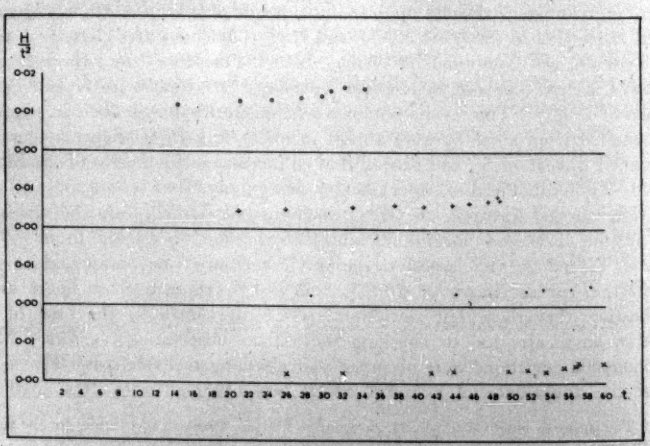

Figure 1, curve *b*. Values of H/t^2 against t.

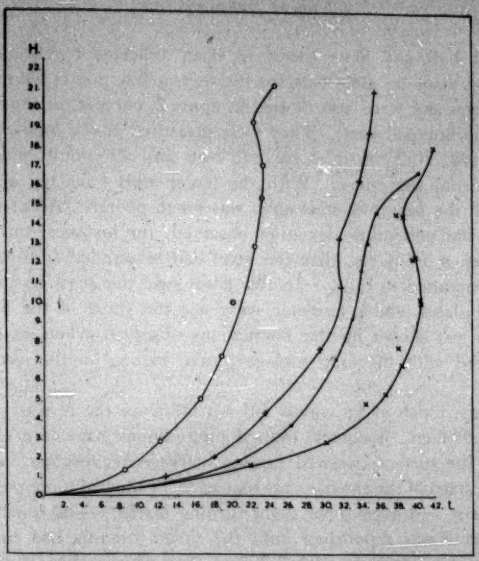

Figure 2, curve *a*. Maximum values of the total bouncing times (t sec.) of Skefko steel balls on long tool-steel anvil T_2 (H_{D50}=720–1016) from heights H cm.

× ball diameter 0·476 cm. + ball diameter 1·000 cm.

● ball diameter 0·794 cm. o ball diameter 1·270 cm.

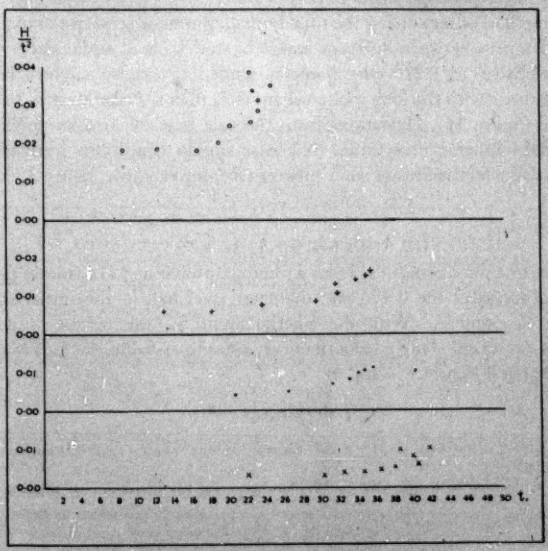

Figure 2, curve *b*. Values of H/t^2 against t.

every size of ball, and have, therefore, been indicated by the curves. With the 0·476 cm. diameter steel ball, for instance, a few points were measured in a separate series and were also plotted in figure 2, curve *a*, to show how closely they fitted the humped part, They were measured at the following H-values: 4·8 cm., 7·7 cm., 10·2 cm., 12·3 cm. and 14·6 cm. No point deviated by more than 3·6 % from the curve. With the larger steel balls the reproducibility obtained with the long tool-steel anvil was much poorer. All of them showed humps, but the maximum deviation observed, for instance, on three curves obtained when a 0·794 cm. diameter steel ball rebounded from the long tool-steel anvil amounted to 15 %. In this latter case the anvil was supported on various base plates which, however, were not the cause of the lack of reproducibility, as was shown by the fluctuations observed when six (H, t) curves were measured with the long tool-steel anvil resting on the same base plate throughout.

The surface finish of the anvils did not influence the results obtained with spheres of 0·794 cm. diameter, even if microscopic inspection (magnification 90 times) of the surfaces showed turning marks. Microscopic inspection had shown at one stage of the experiments that anvil T_1 showed more polishing marks than the others, and even some slight turning marks. The centre part of the polished surface was repolished until the turning marks had been removed. The (H, t) curves obtained with a 0·794 cm. diameter steel ball before and after repolishing were identical. With " extra hard-etched Hultgren steel balls " of 1·000 cm. diameter * rebounding from the short tool-steel anvil, the total bouncing times were shorter before than after repolishing.

The method of measuring the total bouncing time is sensitive enough to show small differences between different makes of steel balls of equal size. A " Hoffman steel ball " of 1·270 cm. diameter tended to exhibit slightly larger total bouncing times with the long tool-steel anvil T_2 than a " Skefko steel ball " of the same size (figure 3). Deviations from the rule that the smaller sphere has the larger total bouncing time under otherwise similar conditions were regarded as a sign of the defectiveness of such spheres (see, for example, figure 3, curve *c*).

§ 5. THE COEFFICIENT OF RESTITUTION

Values of e were computed from a photographic film † (16 frames per second) of the rebounds of the 0·476 cm. diameter steel ball in the initial stages after releasing the sphere. With the Skefko anvil S_1, the height of consecutive rebounds decreased along a smooth curve, and the e_n-values lie for $H_n \leqslant 16{\cdot}3$ cm. within the band (figure 4, *a* and *b*):

$$0{\cdot}960 \leqslant e_n < 1{\cdot}000.$$

* The author is indebted to Mr. A. W. Carrier (Messrs. Skefko Ltd., Luton) for a supply of these steel balls.

† It is a pleasure to thank Mr. C. W. Newberry for his assistance in making the motion picture.

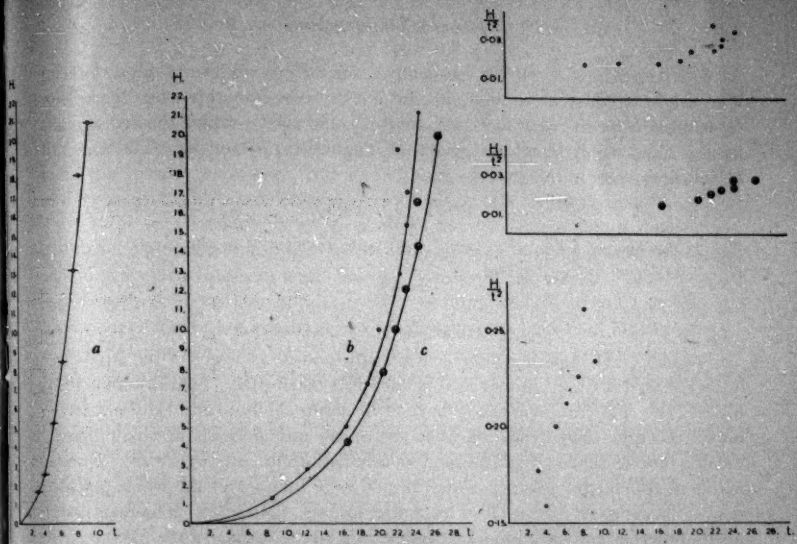

Figure 3. Maximum values of total bouncing times (t sec.) of steel balls of 1·270 cm. diameter on long tool-steel anvil T_3 (H_{D50}=720–1016).
o Skefko ball; -o- defective Skefko ball; ⊙ Hoffman ball.

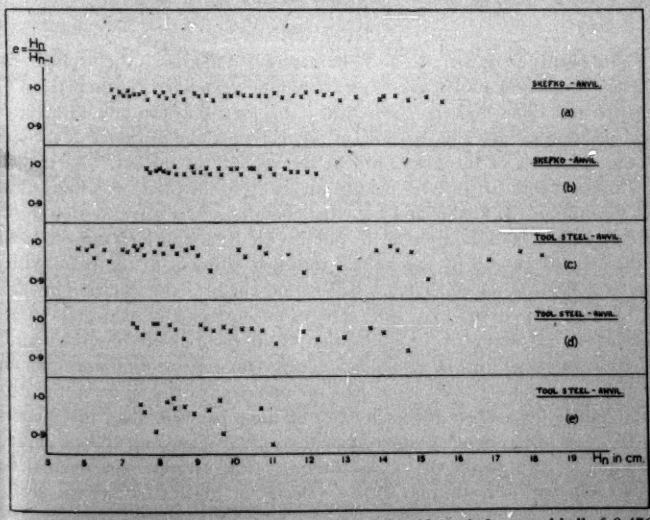

Figure 4. Values of coefficient of restitution of consecutive rebounds for a steel ball of 0·476 cm. diameter rebounding from Skefko anvil S_1 (H_{D50}=875) and from long tool-steel anvil T_3 (H_{D50}=720–1016).

Actually, no value of $e_n > 0{\cdot}995$ was computed in the range $6{\cdot}8$ cm. $\leqslant H_n \leqslant 16{\cdot}3$ cm. The height of consecutive rebounds did not decrease smoothly when the sphere rebounded from the long tool-steel anvil T_2, and the e_n-values showed troughs in this case when plotted a gainst H_n (figure 4. *c*, *d* and *e*). The band of e_n-values is wider in this case:—

$$0{\cdot}875 \leqslant e_n < 1{\cdot}000.$$

Again, the largest value of e_n which had been computed in the range $5{\cdot}9$ cm. $\leqslant H_n \leqslant 19{\cdot}05$ cm. was $e_n = 0{\cdot}994$.

§ 6. LUNETTE-SHAPED MARKINGS

A small grey ring around the centre of the polished surface of the anvils was observed after a great number of rebounds had taken place (Schnurmann, 1939, figure 24). This ring appears to have fairly sharp contours and can be resolved under the microscope (magnification 60 times) into myriads of small sharply defined lunette-shaped marks of random orientation (loc. cit., figure 25). Similar marks in appreciably less dense distribution were found over the whole polished surface of the anvil (for instance its centre, loc. cit., figure 26). The marks were most numerous and of pronounced appearance on the surface of the Skefko anvil. It was found impossible to establish a lower limit for the initial height of fall below which no marks would be produced with the spheres used in the preceding experiments.

§ 7. DISCUSSION

Some twenty years ago, Sir C. V. Raman (1920) discussed the problem of the transverse impact of a solid sphere on an infinitely extended elastic plate of finite thickness and calculated the theoretical coefficient of restitution as a function of the elastic constants and densities of the materials, the diameter of the sphere and the thickness of the plate, and of the velocity of impact. As the result of the impact, annular waves of flexure are set up in the plate, and Raman calculated the sum of the kinetic and potential energies of the wave-motion in terms of the magnitude and duration of the impulse on the simplifying assumption that the disturbance set up by the impact travelled outward in the plate with the velocity of flexural waves having a half-period equal to the duration of impact. The impacts of polished hard-steel spheres on horizontally held glass plates of sufficient size to permit of the application of the theory were observed, and the theoretical and experimental values of the coefficient of restitution were compared. For plates not thinner than about half the diameter of the spheres, the experimental values for e were smaller by two or three per cent than the theoretical values. For plates much thinner than this, experiment and theory agreed in indicating a zero coefficient of restitution. The largest experimental value of the coefficient of restitution with a velocity of impact of 234 cm./sec. was

0·98 when the thickness of the plate was about six times the diameter of the sphere.

The experimental work described above confirms Sir C. V. Raman's theoretical result that the coefficient of restitution for impact on an elastic plate should depend upon the velocity of impact, and should approach unity for very small values of the velocity. This is a consequence of the fact that the duration of impact as given by Hertz's formula varies inversely as the one-fifth power of the velocity, and the radial distance covered by the annular wave diverging from the origin should therefore increase with decreasing velocity of impact. The present experiments at room temperature, however, reveal the important fact that the coefficient of restitution, while tending with decreasing velocity of impact towards a constant value which is close to unity, does not reach unity, not even at the very small velocities of impact which are encountered when a sphere dissipates the whole of its potential energy by a series of consecutive rebounds. Otherwise, the number of rebounds would be infinite, and there would be no sharply defined end-point of rebounding. It so happened that, for instance, a coefficient of restitution of 0·96 was measured (figure 4 *a*) when a sphere of 0·476 cm. diameter struck the anvil S_1 with a velocity of impact of 179 cm./sec., and that a coefficient of restitution of 0·995 was found when the velocity of impact had decreased to 115·5 cm./sec. This latter value was not exceeded during the further decrease of the velocity of impact until the sphere had expended the whole of its initial potential energy by consecutive rebounds. It has been mentioned above that the sphere would come to rest when its potential energy decreased below a certain small value mgH_f.

A priori one would expect to approach the elastic limit without ever reaching it with both the sphere and anvil at room temperature, because an adiabatic compression would alter the temperature of the deformed zone. Locally, this temperature would rise in the case of a material with a positive coefficient of expansion, so that for the duration of an impact, which is of the order of 10^{-5} sec. in this work, a temperature gradient would be set up between the compressed zone and its surroundings. Thus, in addition to the irreversible proportion of the heat of compression, energy would be lost by heat conduction from the deformed zone before its return to the initial condition. A further source of energy loss would be the anharmonic vibrations due to the reflection and interference of both transverse and longitudinal waves in the case of a finite mass of solid. In the neighbourhood of the absolute zero of temperature the adiabatic compression energy tends towards zero, and anharmonic vibrations vanish, so that in the sensibly elastic region of deformation the number of rebounds should become infinite at a temperature sufficiently low for the condition $\lim_{T\to 0} \frac{dA}{dT} = \lim_{T\to 0} \frac{dU}{dT} = 0$ to hold. An appreciable change of the number of rebounds could be expected when the temperature of the sphere and anvil were lowered from room temperature to the temperature of liquid air.

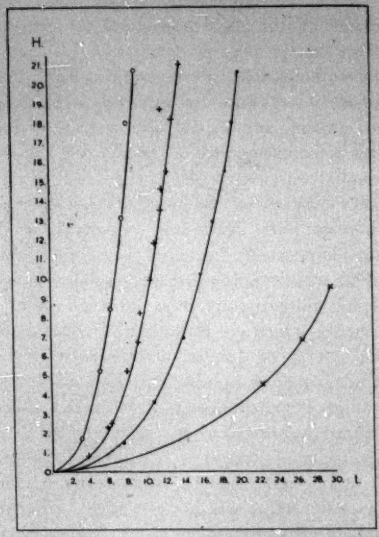

Figure 5 *a*. Maximum values of total bouncing times (t sec.) of Skefko steel balls on short tool-steel anvil T_1 (H_{D50}=498–782).

× ball diameter 0·476 cm. + ball diameter 1·000 cm.
● ball diameter 0·794 cm. o ball diameter 1·270 cm.

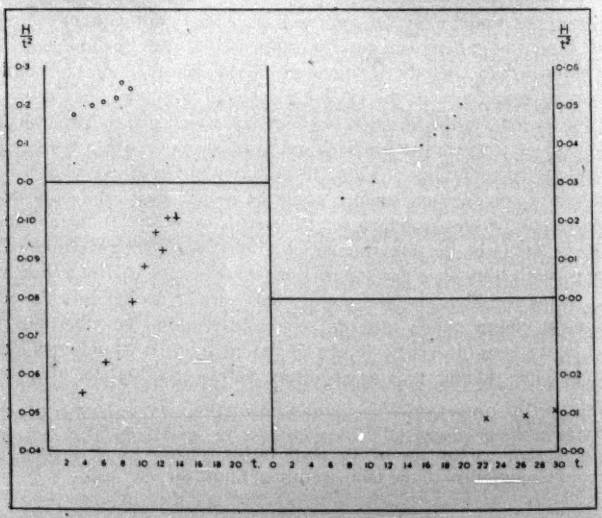

Figure 5 *b*. Values of H/t^2 against t.

The present experiments concern the details of this sensitive method of measuring the total bouncing time at room temperature. Constant values of H/t^2, i.e. of the coefficient of restitution, over an appreciable range of small heights of fall were found when the height of the anvil was at least four times the diameter of the sphere (figure 5 *b*). The duration of impact increases with the diameter of the sphere according to Hertz's formula when the velocity of impact is the same in each case. It is clear from both the (H, t) and $(H/t^2, t)$ diagrams that the coefficient of restitution is larger for the smaller sphere, and it is interesting to note that with a given anvil the range of heights of fall for which H/t^2 is constant decreases with the diameter of the sphere. The former result must be expected even in the sensibly elastic region of deformation, because the rate of energy loss by heat conduction increases with the duration of impact. The latter result indicates the higher sensitivity of experiments with smaller spheres for the energy losses by anharmonic vibrations. Interesting differences were observed between the results obtained with hardened-steel anvils of similar dimensions but different composition (anvils S_1 and T_2). The (H, t) curves obtained with anvil T_2 exhibited humps, and the $(H/t^2, t)$ diagrams (figure 4, *c*, *d*, *e*) showed troughs, so that under otherwise similar conditions the total bouncing times were smaller with this anvil than with anvil S_1. It thus appears that the rate of energy loss from rebounding steel balls depends to an appreciable extent upon the composition of the steel of which the anvil has been made.

The author expresses his thanks to the London Midland and Scottish Railway Company for permission to publish this paper, and to Dr. E. Warlow-Davies for reading the manuscript.

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THE EFFECT OF GRAIN DIRECTION ON THE YOUNG'S MODULI AND RIGIDITY MODULI OF BEECH AND SITKA SPRUCE.

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Received 28 *May* 1941

ABSTRACT. The implications of the assumption that wood possesses rhombic symmetry are discussed, particularly in relation to the effects of grain angle on Young's modulus and rigidity modulus. Simple methods, involving the frequencies of vibration of thin strips cut at the appropriate grain angle, are described to measure these effects. The experimental results agree with the calculated values as closely as can be expected for a variable and elastically imperfect material such as wood.

§1. INTRODUCTION

During the course of a study of the elastic properties of plywood, information was needed on the effect of grain angle on certain elastic constants of solid wood. It was found that the available experimental results were scattered and incomplete, and the investigation which forms the subject of the present paper was therefore undertaken.

The assumption made in most discussions of the elastic properties of wood is that it possesses three mutually perpendicular planes of elastic symmetry (Campredon, 1935 ; Carrington, 1923 ; Hörig, 1931 ; Jenkin, 1920; Price, 1928; Stamer and Sieglerschmidt, 1933). These planes coincide respectively with the longitudinal tangential (LT), the tangential radial (TR) and the radial-longitudinal (RL) planes. Assuming further that wood is homogeneous and perfectly elastic, its elastic properties can be specified by 9 constants (Price, 1928): E_L, E_R, E_T, G_{LT}, G_{TR}, G_{RL}, σ_{LT}, σ_{LR}, σ_{TR}, where, for example, E_L = Young's modulus in L direction, G_{LT} = rigidity modulus corresponding to shear stresses in the T and L directions, $\sigma_{LT} = \dfrac{\text{contraction in T direction}}{\text{extension in L direction}}$ for tension in L direction. In a theoretical paper, Hörig (1931) has pointed out that the transformation formulae originally given by Voigt (1928) for rhombic crystals should apply to the variation of elastic constants with grain direction (θ) in the idealized wood described above.

By reason mainly of its structure, wood is, however, only an approximation to this ideal material. It is neither perfectly elastic (Griffiths and Wigley, 1918) nor homogeneous. The LT surface is not plane, but roughly cylindrical.

The other two surfaces (TR and RL) are more truly plane, but, even in these two cases, there will be some deviations due to local irregularities in the growth of the tree.

Nevertheless, Jenkin (1920) obtained promising results when the calculated effect of θ on E was compared with experiment. As far as is known, no comparison has been made in the case of G, nor are any experimental results available.

The experiments and calculations described later were carried out to provide this comparison, and also to repeat the earlier work on E.

§ 2. THEORETICAL

For simplicity, only the LT plane will be discussed in this section. Equations (1), (2) and (3) below can, however, be used in the other two principal planes (TR and RL) by appropriate interchange of suffixes.

Young's modulus

The formula given by Hörig (1931) for the variation of E with θ is, in the present notation,

$$\frac{1}{E_\theta} = \frac{1}{E_L}\cos^4\theta + \frac{1}{E_T}\sin^4\theta + \left(\frac{1}{G_{TL}} - \frac{2\sigma_{LT}}{E_L}\right)\sin^2\theta\cos^2\theta, \qquad \ldots\ldots(1)$$

where E_θ = Young's modulus at an angle θ to the L direction.

Rigidity modulus

Owing to the fact that the definition of G involves a plane, whereas that of E involves only a direction, the case of G is the more complicated. This can be appreciated by reference to figures 1 and 2, which may, for the moment, be regarded

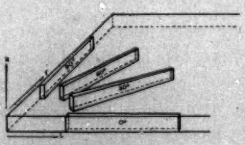

Figure 1.

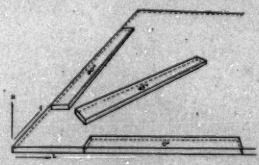

Figure 2.

as representing infinitely thin strips. If shear stresses are imagined to act in the plane of these strips parallel to their edges, it will be seen that there are two ways in which G varies with θ. In the first case (figure 1), θ defines a plane of shearing in which one of the shear stresses is fixed parallel to the R direction, while the other remains in the LT plane, and its inclination with the L direction is given by θ.

The formula giving the variation of this modulus (G_θ') with θ is

$$\frac{1}{G_\theta'} = \frac{1}{G_{RL}}\cos^2\theta + \frac{1}{G_{TR}}\sin^2\theta. \qquad \ldots\ldots(2)$$

The values of G_θ' at 0° and 90° are G_{RL} and G_{TL} respectively; although θ varies in the LT plane, the G_{LT} modulus is not involved at all.

In the second case (figure 2) both the shear stresses are in the LT plane and act at angles θ and $(90°+\theta)$ with the L direction. The formula giving the variation of this modulus (G_θ'') with θ is:

$$\frac{1}{G_\theta''} = 4\left(\frac{1}{E_L} + \frac{1}{E_T} + \frac{2\sigma_{LT}}{E_L}\right)\sin^2\theta\cos^2\theta + \frac{1}{G_{LT}}(\sin^2\theta - \cos^2\theta)^2. \quad \ldots\ldots(3)$$

At both $\theta = 0$ and $\theta = 90°$, this equation gives $G_\theta'' = G_{LT}$.

In both figure 1 and figure 2 the values of E for a given θ should be the same; at 0° and 90° $E_\theta = E_L$ and E_T respectively.

§ 3. EXPERIMENTAL

General

Figures 1 and 2 show the method of cutting the experimental specimens from the original boards. The specimens measured approximately 20 cm. (l) × 2·5 cm. (b) × 0·5 cm. (d), but l was rather variable, and in some cases as low as 10 cm.; the main reason for this was the difficulty of obtaining sufficient length in the R and T directions. The essential differences between the two figures is that in figure 1, the dl face of the specimen lies in the TL plane, while in figure 2 the bl face lies in this plane. The boards of a given species (either beech or Sitka spruce) were all obtained from one consignment. It was not possible to obtain specimens with θ varying in the TR plane, and the investigation was therefore confined to the RL and LT planes. In cutting boards for the latter case, no attempt was made to follow the curvature of the annual rings; all that could be done was to ensure that the rings lay as far as possible in the correct direction relative to the board.

Young's modulus

The values of E were measured by timing the frequency of flexural vibration. The specimen was clamped by one end with its bl face horizontal, a suitable weight attached rigidly to its free end and the period (T) of the resulting vibrations timed with a stop-watch. The method used is, with minor modifications, that described by Worsnop and Flint (1927). The effects of various corrections discussed by Davies (1937) were investigated (Hearmon, 1940); the correction for rotatory inertia of the load was the only one which was ever appreciable. It was found, however, that the approximate expressions of Rayleigh (1877) for the effect of rotatory inertia of the load and of the mass of the specimen were sufficiently accurate, and they are therefore used in preference to the more complicated expressions given by Davies. The final equation for E, which includes the Rayleigh approximations, is, therefore,

$$E = \frac{16\pi^2 l^3}{bd^3T^2}\left(\frac{W}{\{1 - 9K^2/4l^2\}} + \frac{33}{140}M\right), \quad \ldots\ldots(4)$$

where l = length of vibrating part of specimen,
M = mass ,, ,, ,, ,, ,,
W = mass attached to free end of specimen,
K = radius of gyration of W about its line of attachment to the specimen.

Rigidity modulus

Values of G were measured by timing the frequency of torsional vibrations. The specimen was clamped with its length vertical, a bar of moment of inertia I attached rigidly to its lower free end and the period of the resulting vibrations timed with a stop-watch. In order to calculate G from torsion measurements on rectangular specimens of non-isotropic material it is necessary to take into account the ratio d/b and the ratio G_{bl}/G_{dl}, where G_{bl} = rigidity modulus corresponding to shear stresses in the b and l directions of the specimen, G_{dl} = rigidity modulus corresponding to shear stresses in the d and l directions of the specimen. Writing

$$G_{bl} = \frac{4\pi^2 Il}{\mu b d^3 T^2}, \qquad \ldots\ldots(5)$$

μ is given by St. Venant (1864) as

$$\mu = \frac{1}{3} - \lambda \frac{d}{b}\sqrt{\left(\frac{G_{bl}}{G_{dl}}\right)}, \qquad \ldots\ldots(6)$$

while, according to St. Venant, λ is given by the following table:

$\frac{b}{d}\sqrt{\left(\frac{G_{dl}}{G_{bl}}\right)}$	1·20	1·40	1·60	3·0	10·0	∞
λ	0·2006	0·2050	0·2074	0·2100	0·2101	0·2101

In all cases the following method of successive approximations can be adopted to calculate G_{bl}. From experiments on specimens of correct grain orientation, approximate values of G_{bl} and G_{dl} are found by assuming in (6) that $G_{bl} = G_{dl}$. These values can then be used in (6) and (5) to obtain a closer approximation to the true value of G_{bl}. This procedure can be repeated, if necessary, to obtain a still closer approximation to the true value of G_{bl}, but in the majority of cases, this further step is unnecessary.

For a particular θ, and with specimens as in figure 1, $G_{bl} = G_\theta'$; $G_{dl} = G_\theta''$, while with specimens as in figure 2, $G_{bl} = G_\theta''$; $G_{dl} = G_\theta'$. Thus all the data for the calculations described above are obtained from specimens cut as in figures 1 and 2.

Experimental errors

It will be seen from (4) and (5) that the values of E and G are proportional to $1/d^3$. It is therefore particularly important to measure d as accurately as possible. Unfortunately d, which is the smallest dimension, is the most difficult to measure accurately, and it is also subject to the largest errors in machining the specimens. The values of d actually used were the means of 12 separate micrometer measurements along the appropriate part of the strip. Even so, at the

best, d showed a variation along the strip of about 5 %, and at the worst, a variation of over 10 %. Although the mean values of d used to obtain E and G are unlikely to be in error by these amounts, the possibility still exists of rather large errors due to this cause, since a given inaccuracy in d implies three times this error in the final answer.

The possibility of errors due to coupling between torsion and flexure (Brown, 1940) is recognized, but no account is taken of it in the present paper. Preliminary calculation shows that the effect is zero in specimens cut as in figure 1, but may be appreciable for specimens cut as in figure 2. Observation on the vibrating strips themselves indicates that only the torsional frequencies of the latter specimens are affected, if at all, by coupling. Further, these same specimens are affected more than any of the others by the second term in equation (6), so for this reason also comparison between experiment and theory is likely to be least exact for the rigidity modulus of specimens cut as in figure 2

§4. RESULTS AND DISCUSSION

Table 1. Results at approximately 12 % moisture content
Unit = 10^6 dynes/cm^2

(A) Sitka spruce; specimens as in figure 1

θ (degrees)	TL plane				RL plane			
	E_θ obs.	E_θ calc.	G_θ' obs.	G_θ' calc.	E_θ obs.	E_θ calc.	G_θ' obs.	G_θ' calc.
0	74000	81100	6480	6550	76000	81100	6550	5130
15	40900	39700	3180	3220	38000	48300	3960	2860
30	12800	12000	1070	1350	20000	20300	1460	1288
45	4630	4550	535	750	12600	9860	746	735
60	2330	2400	350	520	7060	5990	537	515
75	1790	1680	370	424	5050	4410	364	426
90	1580	1470	360	397	4370	3990	433	397

(B) Sitka spruce; specimens as in figure 2

θ (degrees)	TL plane				RL plane			
	E_θ obs.	E_θ calc.	G_θ'' obs.	G_θ'' calc.	E_θ obs.	E_θ calc.	G_θ'' obs.	G_θ'' calc.
0	80000	81100	5080	5130	94600	81100	7220	6550
30	—	12000	3590	1740	23400	20300	5540	4480
45	6600	4550	2670	1420	14500	9860	4865	4060
60	—	2400	3070	1740	5100	5990	5870	4480
90	1350	1470	3770	5130	3600	3990	5960	6550

(C) Beech; specimens as in figure 1

θ (degrees)	TL plane				RL plane			
	E_θ obs.	E_θ calc.	G_θ' obs.	G_θ' calc.	E_θ obs.	E_θ calc.	G_θ' obs.	G_θ' calc.
0	133000	119000	8450	9750	110000	119000	6800	7620
15	40200	65000	8000	8770	79000	73800	7200	7120
30	18900	29600	5800	6890	45000	38100	6700	6000
45	14100	17600	4560	5320	28900	24600	5800	4940
60	12000	12800	4130	4240	21000	19400	4370	4210
75	10900	10800	3480	3830	17000	17500	4020	3790
90	10900	10300	3750	3660	17500	17000	3570	3660

(D) Beech; specimens as in figure 2

θ (degrees)	TL plane				RL plane			
	E_θ obs.	E_θ calc.	G_θ'' obs.	G_θ'' calc.	E_θ obs.	E_θ calc.	G_θ'' obs.	G_θ'' calc.
0	122000	119000	8470	7620	110000	119000	10600	9750
15	61100	65000	8320	7890	70900	73800	11900	10480
30	27400	29600	8190	8460	40900	38100	11700	12200
45	15300	17600	8100	8780	22800	24600	12000	13400
60	11300	12800	7270	8460	19700	19400	11900	12000
90	9800	10300	7600	7620	16600	17000	10200	9750

The experimental and calculated values of E and G are given in table 1. The experimental figures at 0° and 90° are individual determinations of all the fundamental constants, except σ, required in equations (1), (2) and (3), according to the following scheme:

Specimens as in figure 1				Specimens as in figure 2			
TL plane		RL plane		TL plane		RL plane	
0°	90°	0°	90°	0°	90°	0°	90°
E_L	E_T	E_L	E_R	E_L	E_T	E_L	E_R
G_{RL}	G_{TR}	G_{LT}	G_{TR}	G_{LT}	G_{LT}	G_{RL}	G_{RL}

The averages of the appropriate individual determinations, which were the values used in the calculations, are given in table 2; this table also contains the mean density and the density range of the experimental specimens.

Table 2. Results at approximately 12 % moisture content
Unit = 10^6 dynes/cm^2

Species	Density		E_L	E_R	E_T	G_{RL}	G_{LT}	G_{TR}
	Mean	Range						
Sitka spruce	0·40	0·35–0·48	81100	3990	1470	6550	5130	397
Beech	0·75	0·68–0·84	119000	17000	10300	9750	7620	3660

The effect of σ in any of the equations is invariably small, and the exact values used are therefore unimportant; the values for spruce were obtained from Carrington's results (1923), while those for beech were taken from the paper of Stamer and Sieglerschmidt (1933).

There are a number of factors tending to make the comparison between experimental and calculated values somewhat uncertain. Although all the specimens of a given species were obtained from a single consignment, the intrinsic properties of wood vary from point to point. This is shown both by the figures for density ranges in table 2 and by the variation in values of a given fundamental constant in table 1.

There is the difficulty, discussed in the previous section, of measuring d with sufficient accuracy, and also the difficulty of obtaining specimens accurately cut in the desired direction. In addition to the curvature of the annual rings, the grain in wood may show local deviation of several degrees, and this will be passed on to the specimen, at least over part of its length.

Bearing in mind these limitations, and those set out in § 1, the agreement between experiment and theory shown in table 1 is considered reasonably satisfactory. With one exception (the values of G_θ'' for beech TL, table 1 (D)), the nature of the experimental variation of the constants with θ is given unmistakably by the calculated values. The numerical agreement leaves something to be desired in certain cases, but in the majority it is quite close, and is, in fact, of the same order as that previously obtained by Jenkin (1920) for E. On the basis of the figures given in table 1, it can, however, be concluded that the analogy between wood and a rhombic crystal gives at least a useful first approximation to the variation of E and G with grain angle.

§ 5. ACKNOWLEDGEMENT

The work described above has been carried out as part of the programme of the Forest Products Research Board, and is published by permission of the Department of Scientific and Industrial Research.

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A WHIRLING ARM FOR PRODUCING SMALL DIFFERENCES OF PRESSURE, ITS CALIBRATION BY AID OF A SMOKE MANOMETER AND ITS APPLICATION TO MEASUREMENTS OF SURFACE TENSION

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Received 21 *July* 1941

ABSTRACT. A static-head tube S, as used on aeroplanes, has a closed rounded nose A. If it move, nose forward, in non-viscous air, which is otherwise undisturbed, the pressure at a point C on S, well back from A, equals that at any point in the atmosphere at a distance from C and at the same level. Such a tube, with a small hole at C, is fixed to an arm carried by an electrically driven gramophone table, running synchronously with the 50 cycle/sec. of the "grid" current. A central liquid joint, with which S communicates, allows connexion to be made through S with C. The pressure at a point on the axis of the table is less by Δ_W than that of the atmosphere at the same level and, ideally, $\Delta_W = \frac{1}{2}\rho v^2$, where ρ is density of air and v is velocity of C in its path. The defect Δ_W is tapped off and draws inwards a soap film formed on a circular opening in a bubble cup. The radius r of bubble is found optically.

On account of the viscosity of air, which is significant at slow speeds, and of other deviations from the ideal, the value of Δ_W found with the simple static-head tube, as used in the experiments, does not quite equal $\frac{1}{2}\rho v^2$. A collar on S in front of C serves as a corrector.

A pressure defect Δ_B was produced by a heated buoyancy tube. By adjusting the effective height of this tube, Δ_B was balanced against Δ_W, as given by a corrected static-head tube on the whirling arm. By a smoke manometer, differences between Δ_B and Δ_W of less than 0·1 dyne cm.$^{-2}$ could be detected.

In six comparisons of Δ_W with Δ_B, in which Δ_W ranged from 10·76 to 25·51 dyne cm.$^{-2}$, the ratio Δ_W/Δ_B, where Δ_W, Δ_B are the calculated values, lay between 0·99106 and 1·00749, with 1·00176 as a mean.

Six values of the surface tension of soap solution given by $T = \frac{1}{4}r\Delta_W$, with $r = 4{\cdot}318$ to $r = 10{\cdot}596$ cm., lay between 26·69 and 27·07 and gave $T = 26{\cdot}898 \pm 0{\cdot}045$ dyne cm.$^{-1}$.

§ 1. INTRODUCTION

Differences of pressure up to 25 dyne cm.$^{-2}$ can be produced by aid of a static-head tube S (§ 2) carried round a vertical axis by an electrically driven gramophone table. By a central liquid joint, the pressure defect due to S can be balanced against that due to a heated buoyancy tube or that

due to a spherical soap film. A spherical film of 4·32 cm. radius, formed by a soap solution of surface tension 27 dyne cm.$^{-1}$, gives a pressure difference of 25 dyne cm.$^{-2}$, which is equivalent to a difference of level of 19·70 cm. in air at normal temperature and pressure. Hence, in experiments with bubbles, we cannot afford to deal loosely with differences of level.

§ 2. ELEMENTARY THEORY OF STATIC-HEAD TUBE

If the axis, not necessarily horizontal, of a fixed cylindrical tube S, which has its front end or *nose* closed, be parallel to the general direction of flow of a fluid streaming steadily past it, the pressure of the fluid at a point C on S will, in general, differ from that at any point I at a distance from C and at the same level. But, if the nose be rounded and if C be suitably chosen, the pressure at C will, under ideal conditions, equal that at I.

A tube S (figure 1), with axis AB and of radius a, having a rounded nose at A, is supported at B by a tube T, which connects it to a pressure gauge or other apparatus. When the fluid, which streams steadily past S, is of constant

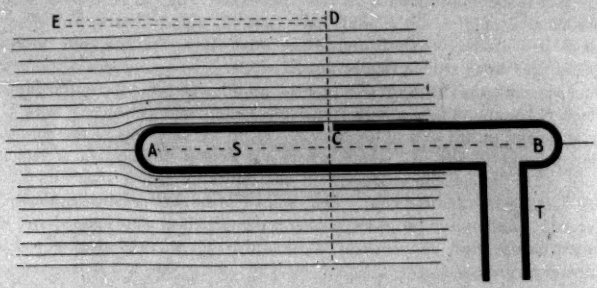

Figure 1. Stream lines near static-head tube.

density ρ and is non-viscous, and when the cylindrical part of S is sufficiently long compared with a, the stream lines have the forms indicated. Let C be a point on S, and let both CA and CB be greater than $6a$. Let CD be perpendicular to AB but not necessarily horizontal. Near the transverse plane containing CD, the stream lines are practically parallel to AB. If a geometrical cylinder of small section be described with CD as axis, the momentum of the fluid within it, when those parts of the stream lines which lie within it are parallel to AB, has permanently no component along CD. Since the fluid is non-viscous, there is no tangential stress at the surface of the cylinder CD. In this ideal case, if p_C, p_D be the pressures at C, D, we have $p_C - p_D = g\rho(z_D - z_C)$, where z_C, z_D are the heights of C, D above a fixed horizontal plane.

If $DC > 10a$, the stream lines through points near D differ very little from straight lines parallel to AB. Hence, any narrow tube of flow, as ED, is of

uniform cross-section, since p and the velocity are constant along ED. Since the fluid is non-viscous, the stress at any point of the surface of the tube ED is normal to that surface and has, therefore, no component parallel to AB. The momentum of the fluid in the length ED of the tube of flow is parallel to AB and is independent of the time, and the rates at which momentum enters ED at E and leaves at D are equal. Hence $p_D - p_E = g\rho(z_E - z_D)$, and thus $p_C - p_E = g\rho(z_E - z_C)$. Hence, the difference of pressure between C and a point E at a distance from C exceeding, say, $10\,a$, and, of course, not near T, depends only on the difference of level and is independent of the velocity of the fluid relative to S.

If there be a small hole in the tube at C, fluid will pass through it until equilibrium is attained. Then, of course, the fluid in S or in any vessel V connected to S is at rest. Hence, since ρ is constant, the pressure at any point P within S or V equals the pressure at points outside S which are at the level of P and are sufficiently far from C.

Relative motion alone is involved, and the problem is unaltered if S advance with uniform velocity through fluid which, but for the motion of S, would be at rest.

In the experiment, the static-head tube moves through air. In general, changes of velocity of air involve changes of pressure and of temperature. But the changes of pressure are less than $[P]/40{,}000$, where $[P]$ is the atmospheric pressure, and no appreciable error is made if the density of the air be treated as constant at points at a given level.

The viscosity of the air is a more serious matter, for now even steady motion calls into play viscous stresses which depend on space-variations of velocity. The pressure at C now differs from that at distant points on the same level, and depends on the velocity v of the static-head tube S relative to the air at a distance from C.

When v is high, as when S is carried by an aeroplane, the viscous stresses are very small compared with the stresses due to the inertia of the air, and the action of the static-head tube is barely affected by the viscosity of the air.

For the low values of v used in the experiment, viscosity has its effect and the pressure at C is somewhat greater than the atmospheric pressure at the level of C. The method of correcting for this and other small effects is described in § 4.

§ 3. THE WHIRLING ARM

Let HKLC (figure 2) be a closed tube of small section, containing air at uniform absolute temperature θ. The tube turns with uniform angular velocity ω about the vertical axis Oz. The parts HK, LC are vertical, HK has Oz for axis and J is any point on Oz between H and K. The horizontal part KL is straight, of section α and length l. The points J, C are higher than KL by z_J, z_C. The pressures at J, K, ... are p_J, p_K, ... If p and ρ be the pressure and density at any point, $p/\rho = R\theta$, where R is the gas constant. In the experiment,

θ is uniform, and $p_C - p_K < p_C/40{,}000$. Hence, to a close approximation, we may consider the air as having constant density ρ. The mass of air in KL is $\rho\alpha l$, and the distance of its centroid G from Oz is $\frac{1}{2}l$. Since G has acceleration $\frac{1}{2}l\omega^2$ towards Oz, the air in KL experiences the force $\rho\alpha l \times \frac{1}{2}l\omega^2$ or $\frac{1}{2}\alpha\rho v^2$ towards Oz, where v is the velocity of L, or of C, in its path. The pressures at L and K produce the force $\alpha(p_L - p_K)$ towards Oz. Hence

$$p_L - p_K = \tfrac{1}{2}\rho v^2 = \tfrac{1}{2}\rho l^2\omega^2. \qquad \ldots\ldots(1)$$

Since KH, LC are vertical, $p_L - p_C = g\rho z_C$, $p_K - p_J = g\rho z_J$. If $z_C - z_J = h$, we have *

$$p_J = p_C + g\rho h - \tfrac{1}{2}\rho v^2. \qquad \ldots\ldots(2)$$

It is easily seen that equation (2) holds whatever the form or section of the tube, provided J be on Oz, the velocity of C be v and the height of C above J be h.

Let, now, C be at the (open) test hole of a static-head tube S, which turns about the vertical Oz. Let the axis of S be horizontal and at right angles to the

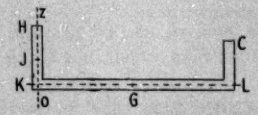

Figure 2. Elementary whirling arm.

perpendicular from C to Oz. Let $[p_J]$, $[p_C]$ be the atmospheric pressures at the levels of J and C. The absolute temperature θ of the air in the revolving tube is assumed equal to that of the atmosphere near the tube. Then, to the approximation implied in (2),

$$[p_J] = [p_C] + g\rho h. \qquad \ldots\ldots(3)$$

Hence, by (2),

$$[p_J] - p_J = [p_C] - p_C + \tfrac{1}{2}\rho v^2. \qquad \ldots\ldots(4)$$

If the conditions were ideal, we should have $[p_C] - p_C = 0$. The "error" $[p_C] - p_C$ for an uncorrected static-head tube, as used on the whirling arm, is negative and of the order of $(\frac{1}{2}\rho v^2)/25$.

If Δ_W be the difference of pressure due to the whirling arm, under ideal conditions,

$$\Delta_W = [p_J] - p_J = \tfrac{1}{2}\rho v^2 = \tfrac{1}{2}\rho l^2\omega^2. \qquad \ldots\ldots(5)$$

Now $\rho = \rho_0 H_N \theta_0 / H_0 \theta$, where θ, p are the absolute temperature and the pressure of the air near the apparatus, H_N and H_0 (= 76 cm.) are the barometric heights, at sea level in latitude 45°, with mercury at 0° c., which correspond (1) to p and (2) to the normal pressure p_0, and, as in § 6, ρ_0 (= 1·29276 × 10^{-3} grm. cm.$^{-3}$) is the density of air at normal pressure p_0 and normal absolute temperature θ_0 (= 273°·2 K.). By § 5, $\omega^2 = 68{\cdot}3491$, and thus

$$\Delta_W = 0{\cdot}158814\ l^2 H_N/\theta. \qquad \ldots\ldots(6)$$

By (9), § 6,

$$H_N = H + 0{\cdot}050 - 0{\cdot}014\,t \text{ cm.}, \qquad \ldots\ldots(6\,a)$$

* When, at each point, we use the true value of ρ, as given by $\rho = p/R\theta$, we obtain, by integration, $p_J = p_C \exp\{(gh - \frac{1}{2}v^2)/R\theta\}$.

where H cm. is the barometric height as observed at the laboratory, and $t°$ C is the temperature of the barometer.

The liquid joint and some other details are shown in figures 3 and 4. The disk W rests on the gramophone table G ; its central hole fits the central pin of the table. To W are soldered a tube E, 4 cm. in diameter, and a pillar F; the top of F is one or two cm. higher than that of E. A tube MM, closed at the ends, passes through both E and F and is soldered to them. The upper

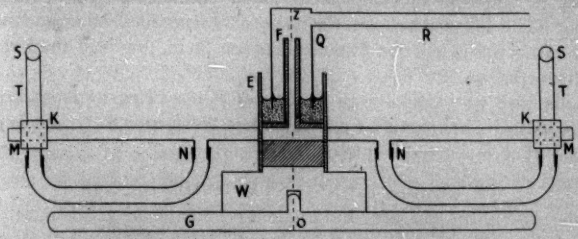

Figure 3. General arrangement of whirling arm.

part of F is 0·7 cm. in diameter. A hole along the axis of F continues into MM. Ideally, E, F are coaxial with the table. From nipples N, N, soldered into MM, rubber tubes pass to the ends of the tubes T, T, which support and lead into the static-head tubes S, S. The tube T is clamped by a split fitting K to the tube MM, which passes through the hole H in figure 4. The axis of S is normal to the plane which contains the axis Oz of the table and passes through

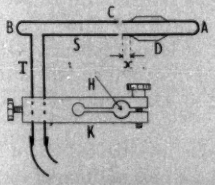

Figure. 4 Static-head tube with correcting collar.

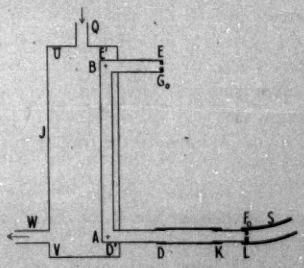

Figure 5. Buoyancy tube.

the test hole C. The length AB of S is 7·4 cm. and CA = 4 cm. ; the diameter of S is 0·64 cm. The axis of S is 7·5 cm. above the surface of G.

Water is placed in the moat between E and the upper part of F. A brass tube Q, 2·4 cm. in diameter, dips into the water; it is soldered to a long horizontal tube R, which is carried by a rod rising from a foot of the tripod base of the motor. A rubber tube joins R to other parts of the apparatus. The diameter of E is

large and that of the upper part of F is small, and thus Q is not near either E or F. If Q be near either E or F, and if one or both of E and F be not coaxial with Oz, periodic changes in the surface of the water between F and Q will be caused by capillary action, and the pressure in Q will suffer small periodic changes. Care must be taken that no water enter the hole in F.

To balance the system, two static-head tubes are used. If each head be adjusted so that its test hole can be brought, by turning the table, to the tip of a pointer mounted on an independent stand, the holes are at equal distances from Oz. The distance between the holes is measured. The mean distance of the holes from Oz is half the distance between them, provided the line joining them intersect Oz.

To diminish any error due to the axis of S not being horizontal, two test holes are used. Their common axis intersects the axis of S at right angles and is vertical when S is correctly placed.

§ 4. CORRECTION OF STATIC-HEAD TUBE

In the experiment, the static-head tube S is under some disadvantages. The velocity is low, and consequently the viscosity of the air has some effect. The swirl of the surrounding air, caused by the rotation of the table, also has an effect. Further, the motion of S is not the ideal one of pure translation along its axis, but S has, in addition, angular velocity about an axis through C parallel to Oz. In the uncorrected static-head tube, $[p_J]-p_J$ (§ 3) is less than $\frac{1}{2}\rho v^2$. It was found that a collar D (figure 4) on S increases or diminishes $[p_J]-p_J$, according as it is in front of or behind C. The diameter of S is 0·64 cm. The length of the collar D is 2·25 cm., and its outer diameter is 0·94 cm. The inclination of its coned ends to the axis of S is 30°. The adjustment of D was effected by aid of a buoyancy tube (§ 6). The slope of this tube was varied until the pressure defect due to it was balanced by that due to the whirling arm. A smoke manometer (§ 7) served as a very sensitive detector of lack of balance. It was found that $[p_J]-p_J$ was equal to $\frac{1}{2}\rho v^2$ when D was in front of C and the distance x (figure 4) between the trailing edge of D and the near edge of the test hole was 0·2 cm.

The behaviour of the whirling-arm apparatus was investigated about 1935 with the help of Mr. C. G. Tilley, my assistant, and the best position of the correcting collar was found. The apparatus, with the collars in this position, was tested, in June, 1941, against the buoyancy tube by Messrs. R. D. Brace, G. J. Dickins and D. H. Wilkinson, with the results given in § 11. The buoyancy tube was efficiently lagged with heat-insulating felt. To obtain closer agreement, some control of the room temperature would be necessary.

§ 5. THE ELECTRIC MOTOR

A Simpson's electrical turn-table is used. Its rotor runs synchronously with the alternating current passing through the stator coils. The rotor is not self-starting, but, if given approximately the correct angular velocity, it soon

falls into step with the current. The stator is mounted on a ball bearing and is capable of a little angular movement. Any departure from its normal position is opposed by a spring. The stator oscillates through one or two degrees while the rotor is falling into step ; when that stage is reached, the stator remains at rest. The speed is independent of the resistance offered to the rotation, until the resistance becomes so great that the rotor drops out of step and ceases to experience a driving torque.

There are 38 poles on the stator and 76 teeth on the rotor. If, when the motor runs synchronously with the 50 cycle/sec. current, the angular velocity be ω, we have

$$\omega = 50 \times 2\pi/38 = 8{\cdot}26735 \text{ radian sec.}^{-1}, \quad \omega^2 = 68{\cdot}3491 \text{ radian}^2 \text{ sec.}^{-2}.$$

The table makes 78·9474 rev. min.$^{-1}$, which is somewhat greater than the standard gramophone speed of 78 rev. min.$^{-1}$.

If the rotation be opposed by a suitable small resistance, such as that due to the motion of the whirling arm through the air, it is possible, by starting the rotor at less than the synchronous speed, to make it run at *about* 62·5 rev. min.$^{-1}$. The stator now vibrates constantly in its bearing. The rotor does not run synchronously but with " slip ", and its speed depends upon the resistance to its motion.

§ 6. THE BUOYANCY TUBE

The buoyancy method of producing small calculable differences of pressure was described by Mr. J. D. Fry (1913) and was at once adopted at the Cavendish Laboratory for use in soap-bubble experiments. It has been used at the National Physical Laboratory for testing very sensitive pressure gauges.

The greater part of the bent tube DD′E′E (figure 5) is surrounded by a steam jacket J. The short parts DD′, EE′ are parallel ; in practice they are horizontal. The tube DE has joints at D′, E′, which are mitred and hard-soldered. The other joints are soft-soldered. The bent tube is put into J, and DD′, EE′ are passed through holes in J and are soldered in place. The caps U, V are soldered to J and the inlet Q to U. The outlet tube W is of the same diameter as DD′ and is coaxial with it. These two tubes act as trunnions. The height of E above D can be varied by turning the system about DW.

The tube D′D is prolonged by a rubber tube DK and a metal tube KL. Into the end of EE′ is soldered a short plug G_0 with a coaxial hole of 0·1 cm. bore; a similar plug F_0 is fitted at L. To be precise, we denote by F, G the *points* in which the planes of the outer ends of F_0, G_0 cut the axes of the holes. A rubber tube S connects L with the smoke manometer or other apparatus. The rubber tube DK ensures that L is very nearly at atmospheric temperature. The diameters of J and of the bent tube are 3·5 and 0·8 cm. respectively; the distance DE is 95·5 cm. The height h of G above F was found by measuring the heights of F, G above the levelled planed edge of a length of small T-girder.

In the theory, the air is stationary both in the atmosphere and in the tube

SDE. Let $[p_F]$, $[p_G]$ be the atmospheric pressures at the levels of F, G. Let p_F be the pressure at F, which is a point in SLK. Let A, B be points on the axis of D′E′ at the levels of F, G. The pressure p_B at B equals that at any point, within or without EE′, which is on the part of the horizontal line BG extending from B through G into the atmosphere, and thus $p_B=[p_G]$. The pressure p_A at A equals that at any point on the horizontal line AF which lies within LD′, and thus $p_A=p_F$.

Let the absolute temperature of the air near the tube be $\theta°$ K., and let that of the steam in J, and so also of the air in D′E′, be θ_S, and let $\boldsymbol{R}$ be the gas constant. Let h cm. be the height of G above F, and so also of B above A. Measuring z upwards, we have, for the atmosphere and for the air in D′E′,

$$dp/p=-gdz/\boldsymbol{R}\theta, \qquad dp/p=-gdz/\boldsymbol{R}\theta_S,$$

and thus, since $p_G=[p_G]$,

$$\log\frac{[p_G]}{[p_F]}=-\frac{gh}{\boldsymbol{R}\theta}, \qquad \log\frac{[p_G]}{p_F}=\log\frac{p_B}{p_A}=-\frac{gh}{\boldsymbol{R}\theta_S}.$$

If the density of air at normal pressure and temperature p_0, θ_0 be ρ_0, we have $1/\boldsymbol{R}=\rho_0\theta_0/p_0$, and thus, if

$$x=gh\rho_0\theta_0p_0^{-1}(\theta^{-1}-\theta_S^{-1}), \qquad \ldots\ldots(7)$$

we have

$$\log\{p_F/[p_F]\}=-x, \qquad p_F=[p_F]e^{-x},$$

and thus

$$[p_F]-p_F=[p_F](1-e^{-x}).$$

In our case, $0<x<10^{-4}$, and we may write $[p_F]-p_F=x[p_F]$. For the factor of x we may, for $[p_F]$, write p, the atmospheric pressure near the apparatus, and then, if the difference of pressure due to the buoyancy tube be Δ_B, we have

$$\Delta_B=[p_F]-p_F=xp. \qquad \ldots\ldots(8)$$

The mass of air at normal pressure and temperature contained in a litre of 1000·027 cm^3, is 1·2928 grm., and thus $\rho_0=1{\cdot}29276\times10^{-3}$ grm. cm.$^{-3}$.

Let σ_0, σ_t be the density of mercury at 0°, $t°$ c. Let gravity be g_N at a point N at sea level in latitude 45° and be g at the Cavendish Laboratory. Let H_N cm. be the barometric height, with mercury at 0° c., at N due to an atmospheric pressure p which gives a barometric height H, with mercury at $t°$ c., at the laboratory.* Then $\sigma_0g_NH_N=\sigma_tgH$. Now $g=981{\cdot}27$, $g_N=980{\cdot}62$ cm. sec.$^{-2}$, $g/g_N=1{\cdot}0006628$, and $\sigma_t=\sigma_0(1-0{\cdot}00018\,t)$. Hence

$$H_N=H\times1{\cdot}0006628(1-0{\cdot}00018\,t).$$

When, as in the experiments, H is nearly 76 cm., we have, with close approximation,

$$H_N=H+0{\cdot}050-0{\cdot}014\,t \text{ cm.} \qquad \ldots\ldots(9)$$

If H_0 (=76 cm.) be the barometric height at N, with mercury at 0° c., due to pressure p_0, then $p/p_0=H_N/H_0$. With $\theta_0=273{\cdot}2$, we find, by (7) and (8),

$$\Delta_B=gh\rho_0\theta_0\frac{H_N}{H_0}\left(\frac{1}{\theta}-\frac{1}{\theta_S}\right)=4{\cdot}5601h\boldsymbol{H}_N\left(\frac{1}{\theta}-\frac{1}{\theta_S}\right). \qquad \ldots\ldots(10)$$

* The small correction to the observed value of H due to the expansion of the barometer scale has been neglected.

§7. THE SMOKE MANOMETER

In 1932, D. A. Wells and W. Lange described *A simple device for demonstrating Brownian movement in gases*. A copy was made for use in lectures at the Cavendish Laboratory. Later, I used a modification of the device as an indicator in balancing the pressure difference due to the static-head tube against that due to the heated buoyancy tube. As a means of showing the Brownian movement of smoke particles, the modified device acts exactly as the original. It is well worth making for the sake of the Brownian movement alone.

The main part of the manometer is a hexagonal block of brass ; its width between opposite faces is 2·54 cm. and its height 2 cm. A central hole A (figures 6, 7) and two other holes B, D are drilled into the block ; each is 5/16 inch in diameter and their axes are in one plane. The holes are coated with a dead black. A transverse hole, 1/8 inch in diameter, starts at K and ends in D. Into B drops a tube S (figure 7) in which are two narrow saw-cuts in a plane transverse to its axis. Light from a *brilliant* horizontal filament is focused on the end of

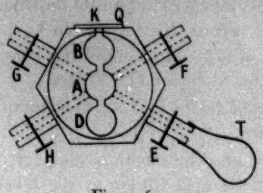

Figure 6.

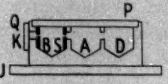

Figure 7

Figures 6 and 7. Smoke manometer.

the hole at K ; the saw-cuts limit the light entering A to a horizontal fan of small depth.* After passing through A, the light enters D and is absorbed by the blackened walls. The top of the cell is closed by a glass plate P, of good optical quality, waxed into a circular recess in the top of the block. The plate may be cut from a spectacle lens of zero power. A small window Q is fixed to the block at K by wax. All the joints must be air-tight.

Four small taps E, F, G, H, such as are used by model makers, are soldered into holes communicating with A. The cell is mounted on a base J (figure 7)

* The apparatus described in the text was made to the specification of Wells and Lange. After the experiments were finished, I removed the tube S and increased the diameter of the hole between B and Q to $^3/_{16}$ inch. It is now easier to illuminate the smoke efficiently. The " background " illumination is increased, but the particles in focus are more brilliant. In making a new manometer, I would abolish hole B and bore a hole $^3/_{16}$ inch in diameter from K into D.

(25 September 1941.) A new manometer has been made. The taps F, H are at the ends of a brass bar $1 \times 1 \times 3\frac{1}{4}$ inches. A hole $\frac{1}{8}$ inch in diameter goes from F to H. The taps E, G are on one face at $\frac{1}{2}$ inch from the ends ; they lead into the axial hole FH. The window Q is on the opposite face. A plain hole from K leads into the central chamber A. Holes B, D are abolished. The long axial hole acts as a reservoir for the smoke, which can drift for some time before it is all swept out of the field of view. Rotherham and Sons, Ltd., Coventry, supplied the taps (No. 54), which have been tested up to 75 lb./inch^2 of air pressure.

by which it is fixed to the stage of a microscope. An objective of one inch focal length suffices. A rubber teat T is fixed to E.

When smoke is to be drawn into the cell, the taps F, H are closed, and E, G are opened. The teat is squeezed, and a smoking match is held near the mouth of G. When T is released, smoke is drawn into the cell. Then E, G are closed. If a wire, which can be raised to red-heat by a current, be placed near and slightly below G, and if a match-stalk rest on the wire, smoke in convenient amount will be produced when the switch is closed.

If the adjustments be good, brightly illuminated particles of smoke in lively Brownian movement will be seen on looking through the microscope. The particles gradually fall to the floor of the cell, but some will be visible even after two or three minutes.

Rubber tubes, not of small bore, fixed to F and H, connect the cell to two sources of pressure. In our case, one is the static-head tube on the whirling arm, and the other is the heated buoyancy tube. If H be opened, and then F be opened (cautiously), the smoke will drift along FH at a rate depending upon the difference of pressure. If this exceed two or three dyne cm^{-2}, the smoke will be swept out of the cell in a fraction of a second, unless the connecting tubes be of small bore. With practice, it is easy to detect a difference of 0·1 dyne cm^{-2}. This is equivalent to a difference of level of $1{\cdot}02 \times 10^{-4}$ cm. in water or of 0·079 cm. in air at normal temperature and pressure.

To test the cell, three taps are closed and the fourth is connected to a source of pressure which differs from that of the atmosphere at the level of the cell. If, when the fourth tap be opened, the smoke drift, the cell is not air-tight. The fault may lie in one or more of the three taps.

§ 8. THE BUBBLE HOLDER

By optical methods it is possible to measure the two principal curvatures at any point of a soap bubble. The work is greatly simplified if the bubble be spherical. If the pressure difference be constant, and if the mass of the film be negligible, the bubble will be spherical only if it spring from a curve which lies on a sphere. The simplest such curve is a circle. To one end of a short tube A (figure 8) is soldered a plate B with a nozzle W. A rubber tube I joins W to the tube R (figure 3) of the whirling arm. To the other end of A is soldered D, an annular plate of brass two or three mm. in thickness. The tube A is held in a lathe chuck and the face E of D is turned to be as nearly as possible plane, and the inner rim is turned to a conical form. The circular edge G must be sharp and clean. It may be slightly smoothed with fine emery paper.

Let O be the centre of G and ON normal to the plane of G. Let HG, a generator of the cone, meet ON in K. Let C be the centre of curvature of a spherical bubble GM based on G. In the experiment, OC is horizontal. The bubble will always be *stably* based on G, and will not retreat to the inner face F of D, if OK be less than the least value of OC which can occur. In the apparatus,

OG = 2·0 cm., and the least values of GC and OC are 4·0 and 3·46 cm. The angle HKO is 45° and OK = 2·0 cm. Hence stability is amply secured.

Two carriages, the bubble carriage and the prism carriage (§ 9), slide geometrically on a pair of horizontal rails attached to a suitable base. The nozzle W of the bubble cup passes through and is clamped in the hole H (figure 9) in the block X, and X is clamped to the horizontal rod Y, which is clamped to the vertical rod Z rising from the bubble carriage. In the ideal position of the cup, the normal ON is parallel to the rails and passes through the point J (figures 8 and 11), which is fixed relative to the prism carriage. The adjustment is easily made by aid of a simple device. A disk fitting into G is fixed to a larger disk and to a coaxial rod. If, when the larger disk is pressed against the face E, the axis

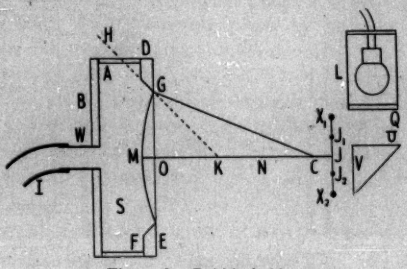

Figure 8. Bubble holder.

of the rod be (1) parallel to the rails and (2) pass through the point J (§ 9), the cup is correctly placed. The correct positions of X on Y and of Y on Z can then be ensured by suitable distance pieces Y_0, Z_0.

If a plate wetted with soap solution be drawn slowly across the opening, a film is formed. If, when the film has closed the opening, the pressure at any level in the cavity S be less than that of the atmosphere at the same level, the bubble will bulge inwards. If the temperature of the air in S equal that of the surrounding air, the pressure difference is very nearly constant over the whole film, which may then be treated as spherical, provided the mass of the film be negligible.

If, at the point M, where the film meets the horizontal axis OC, the external and internal pressures be p_M, q_M dyne cm.$^{-2}$, and if ρ_M be the density of the external air at M, we have, for the pressure difference at height z above M,

$$p_z - q_z = (p_M - q_M) \exp(-g\rho_M z/p_M) = (p_M - q_M)(1 - g\rho_M z/p_M + \ldots).$$

In the experiment, $|g\rho_M z/p_M| < 2{\cdot}6 \times 10^{-6}$, and thus, over the whole film, the pressure difference $p - q$ may be taken as equal to $p_M - q_M$.

If the radius of the spherical film be r cm., and if the surface tension of the solution be T dyne cm.$^{-1}$, we have

$$p - q = 4T/r. \qquad \ldots\ldots(11)$$

§ 9. OPTICAL APPLIANCE

The radius CG (figure 8) of the film is found by optical reflexion. The centre C is self-conjugate. The reflecting power of the film is small, particularly when it is thin, and good illumination is necessary. A right-angled prism UV (figures 8, 10, 11) is placed near C with the face U horizontal and the face V perpendicular to OC. A lantern L, with its axis vertical, contains a small electric lamp, such as a head-lamp bulb, which sends light to U. The window Q, of ground glass, diffuses the light. The prism carriage bears L and UV. Two points J_1, J_2, in a vertical line, are fixed, as described below, relative to this carriage. When the bubble cup is adjusted on its carriage, the normal ON

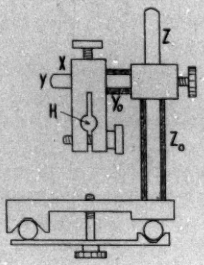

Figure 9.
Bubble carriage.

Figure 11.
Prism.

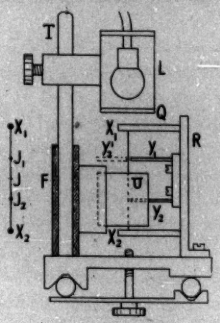

Figure 10.
Prism carriage.

meets J_1J_2 at its mid-point J. Then, by sliding the prism carriage on the rails, the point J can be made to coincide with C.

The presence of the prism makes a special device necessary. From an upright R (figure 10), rising from the prism carriage, project arms X_1, X_2; between them a fine vertical wire X_1X_2 is stretched. From a plate, adjustable on R, project two horizontal steel wires Y_1, Y_2, of which Y_1 is above and Y_2 below the level of the horizontal face U of the prism (figure 10). The wires are in a plane parallel to V, and their free ends nearly touch X_1X_2. The axes of Y_1, Y_2 intersect the axis of X_1X_2 in J_1, J_2; the mid-point is J. The fitting F, which carries the prism, can turn about the pillar T, which supports the lantern. Figure 10 shows the arrangement as seen by an eye placed to the right of the prism in figure 8 and looking towards O. In figure 10, for the sake of clearness, the rod R is shown on the right of the prism. Actually, for convenience, R is on the same side of the carriage as the pillar T, and the arms X_1, X_2 and the wires Y_1, Y_2 project from R towards the right and not, as in figure 10, towards the left.

When the bubble and prism carriages are suitably adjusted, the axis of Y_2', the inverted image of Y_2 due to reflexion at the film, can be made to coincide with the axis of Y_1. The image of that part of the wire X_1X_2 which is below the plane of U lies along the upper part of X_1X_2. Then the image of J_2 is at J_1, and J is the self-conjugate point. The parallax test is used. If the adjustment of the cup be nearly but not quite exact, slight angular motions of the block X and the rod Y (figure 9) will give the coincidences. The best imagery is obtained when the cup is nearly in ideal adjustment, because the best optical use is then made of the central part of the film. Small drops of solution collected on the rim may cause " errors " of form of the film near the rim.

The fitting F is now swung round T to get the prism out of the way, and the distance from J to a point on G (figure 8), which is r, the radius of curvature, is measured. A scale, with a suitable notch for locating G, is used. To diminish any error due to misjudging the position of J, the point on G should be nearly at the level of J.

The parallax test shows that in some bubbles there are small but definite departures from sphericity. In some cases the bubble is not symmetrical with respect to a vertical plane containing ON. If a bubble survive for a comparatively long time, the " defect " disappears.

§ 10. CALCULATION OF SURFACE TENSION

When the film has reached its steady spherical form, the pressure difference due to it equals that due to the whirling arm. The latter difference is Δ_W. If the collars on the static-head tubes be in correct adjustment, we have, by (5), § 3, $\Delta_W = \frac{1}{2}\rho l^2\omega^2$. Since $p-q=4T/r$, we have $T=\frac{1}{4}r\Delta_W$. Thus, by (6) and (6 *a*), § 3,

$$T=0{\cdot}039704\,rl^2(H+0{\cdot}050-0{\cdot}014\,t)/\theta, \qquad \ldots\ldots(12)$$

where H cm. is the barometric height in the laboratory, $t°$ C. is the room temperature, and $\theta=273{\cdot}2+t$. It is assumed that the temperature of the barometer is $t°$ C.

§ 11. TEST OF WHIRLING ARM

Messrs. Brace, Dickins and Wilkinson, using the smoke manometer, balanced the pressure difference Δ_W, due to the whirling arm, against Δ_B, the difference due to the buoyancy tube. By (6), § 3, and by (9), (10), § 6,

$$\Delta_W=0{\cdot}158814\,l^2H_N/\theta, \quad H_N=H+0{\cdot}050-0{\cdot}014t, \quad \Delta_B=4{\cdot}5601\,hH_N(1/\theta-1/\theta_8).$$

Here H cm. is the observed barometric height in the laboratory, $t°$ C. is the room temperature, θ is the absolute temperature of the air near the apparatus, l is the length of the arm, h is the difference of level between the test holes of the buoyancy tube, and θ_8 is the absolute temperature of the boiling point of water under the pressure corresponding to H_N. The boiling point was taken as 100·15° c.,

corresponding to $H_N = 76{\cdot}415$ cm.; then $\theta_S = 373{\cdot}35$ and $1000/\theta_S = 2{\cdot}6785$. The value $H_N = H - 0{\cdot}18$ cm. was used.

l	t	θ	$1000/\theta$	H	h	Δ_W	Δ_B
(cm.)	(°C.)	(°K.)		(cm.)	(cm.)	(dyne cm.$^{-2}$)	
24·61	15·0	288·2	3·4698	76·61	93·32	25·51	25·74
23·41	17·2	290·4	3·4435	76·58	85·87	22·90	22·89
22·50	15·2	288·4	3·4674	76·60	77·27	21·30	21·24
20·675	15·4	288·6	3·4650	76·60	65·50	17·98	17·95
18·48	16·8	290·0	3·4483	76·59	52·90	14·29	14·19
16·05	17·4	290·6	3·4412	76·59	40·20	10·76	10·68

§ 12. MEASUREMENTS OF SURFACE TENSION

Messrs. Brace, Dickins and Wilkinson found the surface tension of a soap solution, using the whirling arm. The radius, r, of the film was measured with the apparatus of § 9. For each value of l, the radius of the path of the test holes of the static-head tubes, five readings of r were taken, each with a fresh film. There were sometimes discrepancies of one or two per cent among the five values. The mean r is recorded. The surface tension was found by (12), § 10, and (9), § 6. Thus

$$T = 0{\cdot}039704 r l^2 H_N/\theta, \quad H_N = H + 0{\cdot}050 - 0{\cdot}014t,$$

where H is the observed and recorded barometric height, t° c. is the room temperature, and $\theta = 273{\cdot}2 + t$. The value $H_N = H - 0{\cdot}19$ cm., corresponding to $t = 17{\cdot}42$° c., was used.

l	r	H	t	θ	T
(cm.)	(cm.)	(cm.)	(° c.)	(° k.)	(dyne cm.$^{-1}$)
24·58	4·318	76·02	17·0	290·2	27·07
22·825	4·950	76·04	17·3	290·5	26·73
21·435	5·666	76·04	17·3	290·5	26·99
19·71	6·674	76·04	17·5	290·7	26·86
17·405	8·626	76·02	17·6	290·8	27·05
15·60	10·596	76·05	17·8	291·0	26·69

The mean value of T is 26·898 dyne cm.$^{-1}$, with a "probable error" of 0·045.

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ELECTRICAL RESISTANCE AND THERMOELECTRIC POWER OF THE TRANSITION METALS

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Received 6 *August* 1941

ABSTRACT. The electrical resistances and thermoelectric powers of the transition metals and their variations with temperature have been measured.

The relation of the results to the theory of Mott and Jones is discussed.

§ 1. INTRODUCTION

The connection between resistance and thermoelectric effect and the energy distribution in the *s*- and *d*-bands of the transition metals has been discussed by Mott (1935, 1936 a and b). The experimental results in this sphere have been rather scanty, and the work described in this paper was undertaken at his suggestion.

As far as electrical resistance is concerned, the position is postulated briefly as follows. In transition metals there is an incomplete *d*- and an incomplete *s*-band. Since conductivity depends mainly on the *s* electrons, the main resistance-producing factor is the $s \rightarrow d$ transition. The large "density of states" at the surface of the Fermi distribution of the *d* electrons causes big probability of $s \rightarrow d$ scattering, and so a big electrical resistance. This explains the high resistances of Ni, Pd and Pt compared with those of Cu, Ag and Au. If the curve relating density of states in the *d*-band to energy is concave to the energy axis, an increase of temperature will reduce the probability of $s \rightarrow d$ scattering, and so the resistance will increase less quickly than for a normal element. For other transition elements for which the *d*-band is filled to a different level, the change of resistance with temperature will depend on the shape of the *d*-band at the top of the Fermi distribution.

For thermoelectric power, theory suggests that if the *d*-band is nearly full, the thermoelectric power will be negative. Again, if the *d*-band is only partially filled, the sign of the thermoelectric power will depend on the shape of the surface of the band at the top of the Fermi distribution.

This work, then, was undertaken in order to provide data for comparison with the many theoretical calculations which have been made on the shape of the bands in transition metals.

§ 2. METHODS OF MEASUREMENT

Most of the elements used were available in the pure form only in small pieces varying from a few millimetres to a few centimetres in length. It was necessary therefore to use methods of measurement suitable for small specimens.

(a) *Resistance measurements*

For the experiments on molybdenum, chromium, vanadium, niobium, tantalum and titanium the apparatus used for resistance measurements was that previously described in connection with experiments on the resistance of nickel. As less accuracy was required than in the nickel experiments, the temperature regulator and one of the copper furnace liners were omitted. The specimens themselves, usually about 1 cm. long, were welded to nickel leads just as the pure nickel of the earlier experiments was welded to leads of ordinary nickel. The specimens could not of course be bent into a U as was done with the pure nickel.

For high-temperature work the specimen and its leads could be mounted in a furnace, and for low-temperature work in a Dewar flask containing liquefied gases boiling at atmospheric or reduced pressures.

In the case of vanadium, the only material available was in the form of irregular pellets. From these we succeeded in producing two specimens of section about 0·6 mm. square and length 6 mm. (4 mm. between potential leads). With such a small specimen it was obviously undesirable to weld direct on to the ordinary 2 mm. thick nickel leads. Instead, the current was led in through two intermediate platinum wires (about 28 s.w.g.) welded to the vanadium (to which it joins very easily) and to the nickel.

(b) *Measurements of thermoelectric power*

The easiest method of obtaining the absolute thermoelectric power for any element is to measure the relative thermoelectric power between the element and some other substance of which the absolute power is known. The only element for which the absolute thermoelectric power is known over a wide range of temperature is copper. But copper could not be used directly as a reference metal because of the difficulty of joining it to the various elements used. The following procedure was therefore employed:—

A thermocouple of pure copper and pure silver was calibrated with various standard melting points. The e.m.f. obtained (figure 16), and therefore, also, the difference of the thermoelectric powers, is small except for temperatures over 500° c. This gave us the absolute thermoelectric power of silver, which, since it welded easily to all our specimens, was used as the reference metal.

The method of experiment is represented in figures 1 and 1 *a*. The specimen (*a*) was spot-welded to nickel rods (*b*) and (*c*) and was heated at one end by radiation from a tungsten spiral surrounded by a radiation shield (*d*). The temperatures at the two ends of the specimen were given by two calibrated silver *vs.* platinum thermocouples (*f*) and (*g*). The thermoelectric power of the specimen

relative to silver was then obtained from the e.m.f. between the two silver leads and the temperature difference between the two silver *vs.* platinum thermocouples. The method of welding on the couples is shown in figure 1 *a*. By having a small space (about 0·3 mm.) between the silver and the platinum we ensure that the measured temperature is that of the specimen. Had contact been made directly between the two thermocouple leads, we could not have been certain that the junction temperature was not appreciably less than that of the heated specimen. A radiation shield (*l*) prevents radiation falling directly on the thermocouples.

For temperatures below 0° c. different temperatures and temperature gradients were obtained by immersing the rod (*c*) and the specimen to variable depths in liquid hydrogen, nitrogen, oxygen, ethylene or propane. In some cases solid carbon dioxide mixed with acetone was also used.

(c) *Combined resistance and thermoelectric measurements*

One disadvantage of the methods described above lies in the necessity of transferring the specimen from one apparatus to another between making the resistance and thermoelectric measurements. The danger of damage to specimens was minimized by cutting the nickel rods near to the specimens rather than cutting the specimens themselves, thus avoiding any heavy spot-welding of the latter. It was, however, almost always necessary to re-weld the thermocouples. Later, an apparatus was devised which enabled us to make resistance and thermoelectric measurements at both high and low temperatures with only minor alterations to the apparatus, and involving no interference with the specimen. In this apparatus a greater convenience in use was secured without serious loss in accuracy. Both forms were used in obtaining the results given below.

In this later apparatus, the wire-wound vacuum furnace was disposed of. The heating was provided by two radiation furnaces (*a*) and (*b*) (figure 2) symmetrically placed with respect to the specimen (*c*). The heater (*a*), supported by its own leads, was mounted at one end of the glass envelope, heater (*b*) being carried by the opposite end. The two portions of the glass vessel were fitted together by ground flanges (*d*) (*d*) made air-tight with plasticene, and the apparatus was evacuated to below 10^{-4} mm. mercury.

For resistance measurements, the current in the specimen was carried by nickel rods (*l*) (*l*) (*l*), which passed through water jackets (*f*) (*f*). An air-tight seal was made at (*g*) with sealing wax. (*h*) is a small molybdenum spring to allow for thermal expansion.

The apparatus was easily dismounted for changing specimens, etc. For this purpose the junction (*g*) was softened and the lower part of the glass apparatus, together with the heater (*b*), was withdrawn.

For resistance measurements, the heaters were run so as to give the same temperature at the two thermojunctions (*m*) (*m*), the silver leads of the thermocouples being used as potential leads. Equality of temperature at the two ends

of the specimen does not of course imply uniformity along the specimen. There will certainly be a minimum temperature between the junctions, and so the measured temperature will be a little above the average value for the specimen.

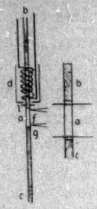


Figure 1 *a*.

Figure 1.

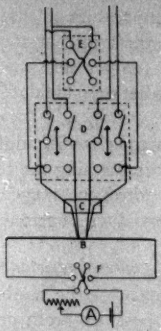


Figure 3.

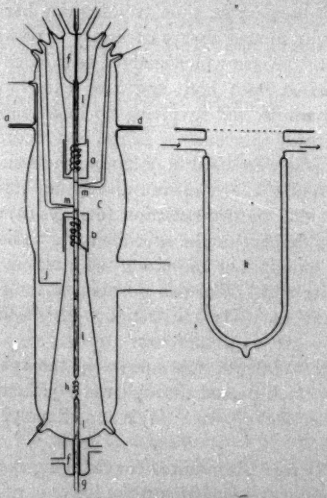


Figure 2.

This may be detected and, if necessary, corrected for as follows :—

A {resistance, temperature} curve is obtained with, say, 8 mm. between the junctions. Then the distance is reduced to 4 mm. and the measurements

repeated. In the second case, the minimum will be less pronounced, and the measured temperatures will therefore approach more closely to the average value for the specimen. This will result in the second curve being shifted slightly with respect to the first curve. Now it may easily be shown that provided the " minimum " is not pronounced, its depth will be proportional to the square of the distance between the thermojunctions. Thus three-quarters of the error is removed by halving the distance between the junctions. One can thus estimate accurately the ideal curve.

For good conductors, such as iridium and rhodium, the error due to this minimum was negligible, but for bad conductors like zirconium and tantalum it amounted to a few degrees. The values shown in the table on p. 702 have been corrected where necessary.

For thermoelectric measurements, the heaters were run unequally. Obviously, in theory, any temperature and any temperature gradient are obtainable in this way.

For measurements at low temperatures, the lower end of the glass vessel, with the heater (*b*), could be removed and the nickel lead cut just above the molybdenum spring. The lead (*j*) was then spot-welded to the nickel rod so as to become one of the current leads. A Dewar flask (*k*) with ground flange could be put into position and the specimen wholly or partially immersed in a liquid boiling at atmospheric or reduced pressure. In this way temperatures down to 14° K. were reached (liquid hydrogen at reduced pressure). Complete immersion of the specimen—giving uniform temperature—was of course needed for resistance measurements. Temperature gradients in the specimen could be obtained by using a lower liquid level, and, if desired, could be increased by using the heater (*a*) in addition.

The electrical connections allowing measurements of the two junction temperatures, the thermoelectric force (silver—specimen—silver), and the potential drop in the specimen due to the current (and thus the electrical resistance) are shown in figure 3. The current in the specimen was measured by the substandard ammeter A. The absolute accuracy of the instrument was immaterial, as the same current served to produce the potential throughout any particular set of readings. Readings of current could be repeated to at least one part in 3000.

The specimen is shown at B (figure 3) and the cold junctions of the thermocouple at C. D represents a four-pole double-throw mercury switch. When in the " up " position, the thermocouple e.m.f.s are measured. When in the " down " position, the potential due to the current superimposed on that between the silver leads due to accidental temperature gradient in the specimen is measured. To eliminate this residual thermal e.m.f., and to measure the e.m.f. produced by the current, a pair of reversing switches E, F were used so that the e.m.f. produced by the current could be reversed with respect to the thermo-e.m.f. The magnitude of this stray e.m.f. will depend on the currents in the radiation furnaces. When resistance measurements are being made, the furnace currents

are adjusted to give as little thermo-e.m.f. as possible, thus indicating equality of temperature at the two ends of the specimen.

The Tinsley thermocouple potentiometer which was used to measure the e.m.f.s was designed to deal with only two external e.m.f.s. The leads from the

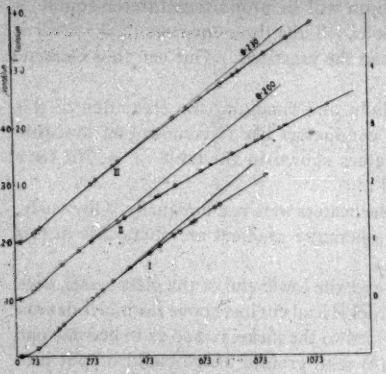


Figure 4.
I. Vanadium. II. Niobium. III. Tantalum.

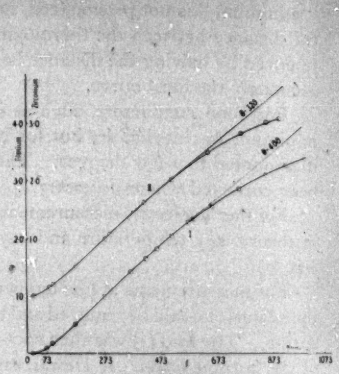


Figure 5.
I. Titanium. II. Zirconium.

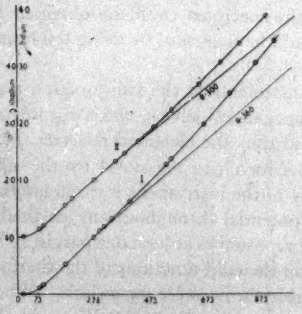


Figure 6.
I. Rhodium. II. Iridium.

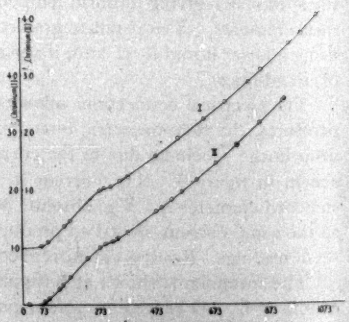
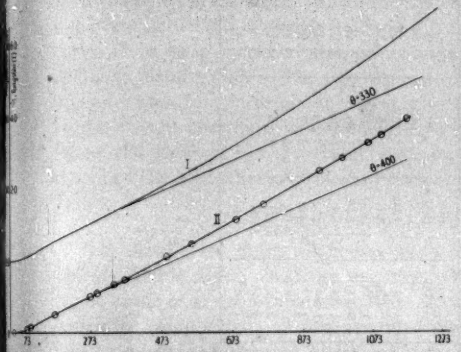


Figure 7.
I. Chromium annealed at 600° C.
II. Chromium outgassed just below melting point.

reversing switch E (figure 3) were therefore tapped into two already existing leads to the potentiometer. This causes no practical difficulty, as switch D prohibits any superposition of e.m.f.s.

§ 3. RESULTS

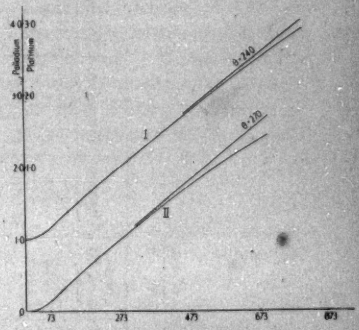
The results of the resistance measurements are shown in figures 4 to 9 and in the table on p. 702, which shows the value of $\frac{R-z}{R_0-z}$ as a function of the absolute temperature T ; R is the resistance, R_0 the resistance at 0° C. and z is the residual resistance at absolute zero. All values of R, R_0 and z are potentiometer readings, instead of resistances in ohms, and so are in arbitrary units depending on the current employed. The values at 14° K., 20° K., 77° K. and 90° K. represent actual experimental points, but the higher temperature values are interpolated from the graphs. The actual experimental values are plotted on these graphs. A rough estimate of the specific resistance at 0° C. (ρ) is given for metals for which this quantity is not well known. It has of course been calculated after allowing



Temperature (° K.)

Figure 8.

I. Tungsten. II. Molybdenum.



Temperature (° K.)

Figure 9.

I. Platinum. II. Palladium.

for the residual resistance, so that it represents an estimate of the specific resistance of a pure specimen.

As predicted theoretically, the resistance does not become a linear function of temperature at high temperatures, as is approximately the case with non-transition metals. (At high temperatures the resistance of ordinary metals usually increases slightly faster than the first power of temperature, due probably to a progressive change in the characteristic temperature Θ due to expansion of the metal.) The peculiarity of the transition metals has been emphasized by drawing on each {resistance, temperature} graph the resistance calculated from the Grüneisen formula. The characteristic temperature was chosen to give the best fit between experiment and the Grüneisen function at low temperatures. (According to Mott's theory, the peculiar behaviour of the transition metals should be manifested only at high temperatures.) The resistance curves are

$$\frac{R-z}{R_0-z}$$

T (°K.)		Vanadium	Tantalum I	Tantalum II	Niobium I	Niobium II	Niobium III	Titanium	Zirconium	Rhodium	Iridium	Chromium I	Chromium II	Molybdenum
	R_0	455	284·5	304	1872	2752	2051	1279	286	112·5	126	234	186	439·5
	z	148	13	31	290	550	585	327	11	0	0	4	3	52·5
	R_0-z	307	271·5	273	1582	2202	1466	952	275	112·5	126	230	183	387
	ρ	$18{\cdot}2\times10^{-6}$	$12{\cdot}8\times10^{-6}$	—	—	—	—	83×10^{-6}	$17{\cdot}1\times10^{-6}$	—	—	—	—	—
14		0	0	—	·002	·001	—	0	—	—	—	—	—	0
20		·005	·002	—	·006	·005	·006	·001	0	0	0	·004	—	·001
77		·142	·196	—	·199	·192	—	—	·160	·11	·135	·058	—	·096
90		·195	—	·245	·257	·250	·246	·180	·207	·16	·19	·104	·104	·148
173		·570	·600	·600	·620	·620	·620	·53	·55	·54	·58	·50	·495	·53
273		1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000
373		1·39	1·38	1·38	1·345	1·345	1·340	1·445	1·46	1·48	1·42	1·26	1·235	1·48
473		1·76	1·76	1·76	1·645	1·645	1·645	1·89	1·89	1·97	1·85	1·62	1·60	2·00
573		2·105	2·12	2·12	1·945	1·945	1·945	2·305	2·29	2·47	2·29	1·995	1·98	2·52
673		2·43	2·47	2·47	2·22	2·22	2·22	2·64	2·63	2·99	2·77	2·405	2·365	3·04
773		2·745	2·805	2·805	2·485	2·485	2·485	2·95	2·905	3·56	3·26	2·845	2·795	3·58
873		3·045	—	3·14	2·74	2·74	2·74	3·20	3·10	4·11	3·76	3·310	3·23	4·14
973		—	—	3·47	3·00	3·00	3·00	3·40	—	—	—	—	3·685	4·72
1073		—	—	3·78	—	—	3·245	—	—	—	—	—	4·18	5·28

seen to be sometimes concave and sometimes convex to the temperature axis but, with one exception, to be similar for elements with the same electron configurations.

This one exception is the case of chromium, which has the same electron configuration as molybdenum and tungsten. This material was of the highest purity (99·99 %), but behaved in a manner quite unique for a pure metal. The extraordinary shape of the curve around 0° c. is most striking, and there is a counterpart in the thermoelectric power. The {resistance, temperature} curve is not of the form which would be expected from a phase change. The value of the resistance at 0° c. is independent of whether the previous reading was taken at 100° c. or at −183° c. After an initial heating to 600° c. there was always, in addition, a general decrease in the resistance of chromium (probably due to an increase in grain size). Subsequently the {resistance, temperature} curve was quite stable and reversible. Curve I is for a specimen annealed at 600° c., curve II is for a specimen strongly outgassed by electron bombardment just below its melting point. The two curves are nearly identical. We have, unfortunately, been unable to obtain high-purity chromium from any other source. In the case of chromium of commercial purity, no sign of the peculiarity was found, but this was not surprising, since the residual resistance was large and swamped the temperature-dependent component of resistance.

§ 4. THERMOELECTRIC POWER

The results are shown in figures 10 to 15, in which thermoelectric power (microvolts per degree) is shown as a function of temperature. Viewed over the whole range of temperatures from 20° K. upwards, the results will be seen to be somewhat complicated. At temperatures above (say) 100° c. the curves—except for chromium—are not very dissimilar from those for ordinary metals, for which it is well known that the thermoelectric power is almost a linear function of temperature. Special reference should be made to the thermoelectric powers of titanium and niobium. Owing to their poor thermal conductivity, there was considerable temperature difference between the junctions. This, combined with the marked curvature of the {thermo-e.m.f., temperature} curves, made it difficult to estimate thermoelectric powers directly. Consequently, curves were first constructed showing total thermo-e.m.f. as a function of temperature. The thermoelectric powers could then be obtained from the tangents to these curves. The results shown in the figures are of absolute thermoelectric power.

For completeness, a few results are given from the results of other workers. The resistance curves of tungsten (figure 8), palladium and platinum (figure 9) are taken from the *Handbuch der Experimental Physik*. The thermoelectric-power curves for palladium, platinum and nickel above its Curie temperature (figure 15) have been taken from the *Handbuch der Metallphysik*. To distinguish these results from new measurements given in this paper, no points are included on the appropriate graphs.

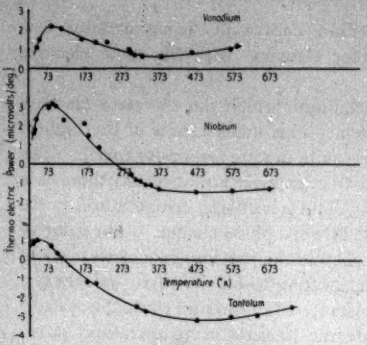


Figure 10.

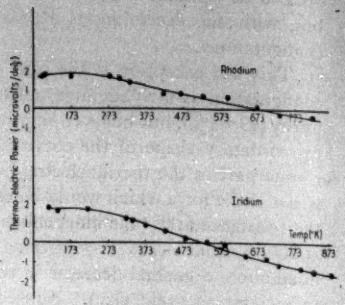


Figure 11.

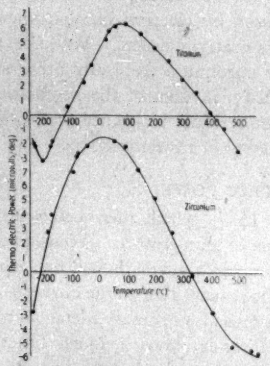


Figure 12.

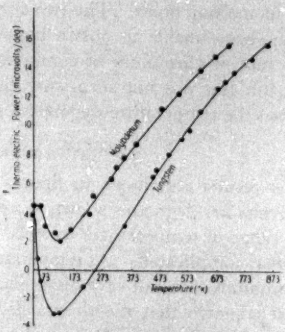


Figure 13.

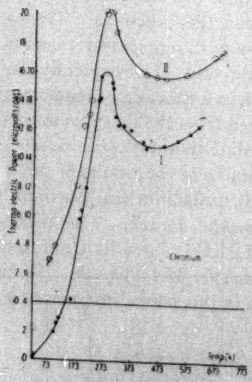


Figure 14.

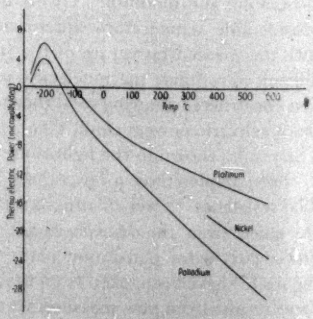


Figure 15.

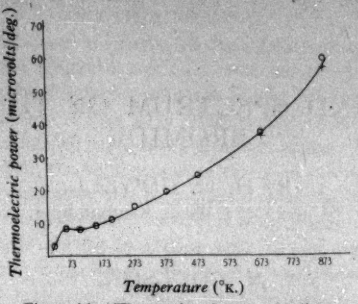


Figure 16. Thermoelectric power of silver.

§ 5. DISCUSSION OF RESULTS

It is evident from the results that neither resistance nor thermoelectric power can be fully described by a simple theory like that evolved by Mott using *d*-bands of simple shape. Neither the variation of resistance with temperature nor the thermoelectric power shows progressive change with the number of electrons in the *d*-band, and so neither can be explained by the shape of the *d*-band at the top of the Fermi distribution unless the band is of complicated shape. We leave it to subsequent theoretical work to show to what extent the results can be explained by a more complicated model.

§ 6. ACKNOWLEDGEMENTS

The writer wishes to express his thanks to Professor Mott for much helpful discussion and guidance, and to the Colston Research Society for a grant towards the expenses of this investigation. Also to Dr. Sykes, of the Metropolitan-Vickers Co., for a specimen of niobium, and to the Vanadium Corporation of America for a specimen of pure vanadium.

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THE BAND SPECTRUM OF THALLIUM BROMIDE

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Communicated by W. Jevons, 9 July 1941

ABSTRACT. The spectrum TlBr has been photographed in absorption and in emission from 3400 to 4500 A. The analysis of the main band system $1 \rightleftarrows {}^1\Sigma^+$ can be represented by the formula

$$\nu = 29191{\cdot}5 + (108{\cdot}32u' - 5{\cdot}15u'^2 - 0{\cdot}22u'^3) - (192{\cdot}10u'' - 0{\cdot}39u''^2),$$

where $u = v + \frac{1}{2}$.

A very small but intense band system occurs, in emission only, at 3950 A. The lower state of this system is the upper state of the main system. Predissociation occurs in both systems, and the unstable levels responsible are the upper levels of continua at 1900 and 3300 A. Other continua occur at 2700 and 2960 A.

§ 1. INTRODUCTION

The absorption spectra of the halides of Tl have been studied by Butkow (1929) and by Butkow and Boizova (1936). In more recent publications, Howell and Coulson (1937 and 1938) have shown that in the cases of TlF and TlCl the analyses submitted by the former workers were incorrect. They further extended the investigation by obtaining the spectra in emission, and were able to show that predissociation occurred extensively. The present paper describes the continuation of this work in a new examination of the spectrum of TlBr in both emission and absorption. For this molecule, Butkow found that with the temperature of the absorbing vapour at 330° C. the discrete band spectrum lay in the region of 3400 A., and by raising the temperature to over 500°, he was able to extend the bands to 3600 A. He fitted 14 red-degraded bands into the following quantum formula :—

$$\nu = 29149{\cdot}6 + (103{\cdot}8n' - 6{\cdot}05n'^2) - (192{\cdot}2n - 0{\cdot}32n^2).$$

This was supported by his measurements on the Br isotope-effect.

In addition, continuous absorption from 3272 to 3387 A. (maximum at 3332 A.) and from 2630 to 2910 A. (maximum at 2690) were reported.

§ 2. EXPERIMENTAL

The absorption tube was of quartz, 30 cm. × 2 cm., with side-limbs which were provided with an electrical heating circuit independent of that of the main tube. A hydrogen discharge lamp running at 0·1 amp., 2500 v. served as a source of continuum. This gave good exposures in 10 minutes on a Hilger E1 quartz spectrograph using Ilford Ordinary plates. As the temperature of the vapour was raised, the band spectrum extended towards the red, as also did the continuous spectrum on the short-wave side of the bands. As a result the continuum completely obscured more and more bands, and so, for a given temperature, only those on the long-wave edge of the continuum could be photographed in a single exposure. By careful temperature regulation, the bands have been photographed in short stages up to nearly 4000 A. The continua have been studied with a Hilger small quartz spectrograph.

The same tube was at first used for the emission work, but later it was abandoned because of much band self-reversal, due to the long vapour-path, and a shorter tube of the central-capillary type was substituted. This was heated in a silica-tube furnace, $1\frac{1}{2}''$ in diameter, wound with lagged Nichrome ribbon, a small aperture exposing the capillary for photographing.

The excitation energy was provided by the $\frac{1}{2}$-kw. oscillator used in previous work (Howell and Coulson, 1937). To prevent H.F. sparking between the leads and the furnace heating-circuit, H.F. chokes were placed in the latter circuit and the leads were enclosed in Pyrex tubing. The spectrum was photographed at various temperatures in a small quartz spectrograph in order to determine the optimum conditions for its production. When the requisite heating currents had been determined, the larger (Hilger E1) instrument was used, and good plates were obtained with exposures of 1 hour. Measurements were made on Ilford Monarch plates using iron-arc standards.

§ 3. DESCRIPTION OF THE SPECTRUM

(a) *Absorption*

The band system reported by Butkow has been extended to almost 4000 A. and the continua at 3330 and 2700 A. have been photographed. In addition, two more continua have been observed. The first begins somewhere below 1900 A. and stretches to longer wave-lengths. The second—a narrow diffuse band—stretches from 3370 to 3392 A., with a maximum at 3380; this can just be detected on Butkow's reproduction. As the temperature rises all these continua extend, particularly toward the red. Because of this, the continuum at 3330 A. obscures that at 3380, and then covers most of the discrete bands, leaving only those on the red edge. There is probably another very narrow and weak continuum covering some of the bands at 3430 A., but not obliterating them.

(b) *Emission*

At the lowest pressures, lines were predominant in the discharge, and as the pressure was increased; impurity spectra appeared, in particular the

OH band at 3090 A. However, these were replaced at higher pressures by the bromide bands extending from 3400 to 4500 A. A very intense group of bands around 3950 A. is a prominent feature of the spectrum. These do not appear to be related in any way to the main system, and they are not found in absorption. Continua also appear in emission, viz. one at 3330 A., corresponding to that found in absorption, a longer one stretching from 2850 to 2990 A., with a maximum at 2960 A., and a third superposed upon most of the band system.

All of the bands measured have been classified in one or other of the two band-systems. Those which are not given in table 1 are presented in table 5.

§ 4. THE MAIN BAND SYSTEM

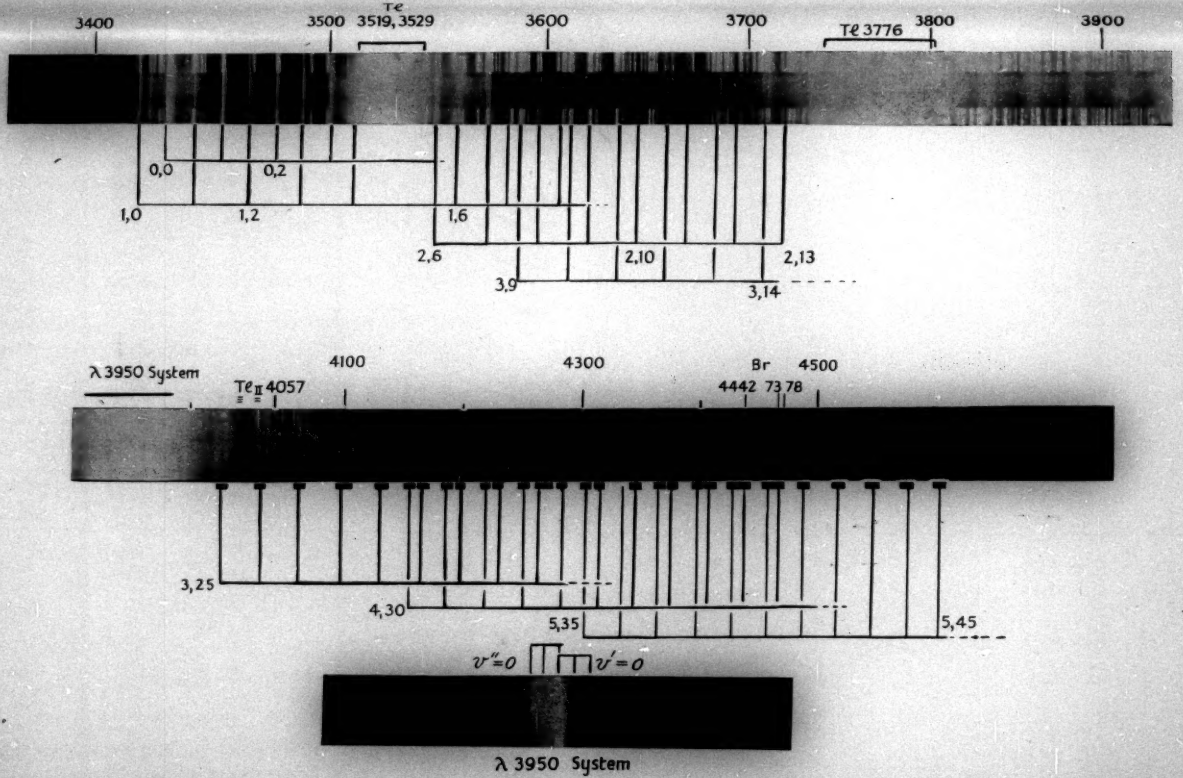
As a photograph of the absorption spectrum has already been published by Butkow (1929), a plate of the emission spectrum only is reproduced here. Whilst the bands near the short-wave side are sharp and line-like, those above 4000 A. are so diffuse that accurate measurements have been found impossible, and estimates of the wave-lengths of the centres of the diffuse heads have been made by interpolating between Fe lines on an enlargement. For the sharp heads, agreement to within 0·5 $cm.^{-1}$ has been obtained between emission and absorption measurements, but the consistency of the values given for the diffuse emission bands from a number of determinations is not better than 5 $cm.^{-1}$.

The intense Tl lines 3519, 3529 and 3776 A., broadened by pressure, hide a number of bands, but in most cases these are also found in absorption, and so these gaps have been filled in. No sign of any rotational structure has been observed in any of the bands. Intensity considerations point to the 0, 0 band being at 3430 A., and this is supported by measurements on the isotope effect; this conclusion is also in agreement with Butkow's allocation. The quantum array of the bands is given in table 1, from which it will be noticed that many of the bands with $v'=2$ fit equally well into the progression $v'=0$. This was noticed by Butkow, who chose the latter alternative. However, if this were correct there would be a surprising absence of $v'=2$ bands; furthermore, the present arrangement is that which is to be expected from the intensity parabola for such a molecule. No bands belonging to progressions higher than $v'=3$ are observed in absorption and the few with $v'=3$ are diffuse. The same bands appear in emission, with the addition of many more on the red end of the system, which are excessively diffuse and which belong to higher v' values. The diffuseness is so great around 3800 A. that even the Br isotope-effect is unresolved. The following equation has been derived for the TlBr band-heads:—

$$\nu = 29195{\cdot}5 + (108{\cdot}32u' - 5{\cdot}15u'^2 - 0{\cdot}22u'^3) - (192{\cdot}10u'' - 0{\cdot}39u''^2),$$

where u' and u'' are written for $v'+\frac{1}{2}$ and $v''+\frac{1}{2}$.

Apart from the cubic term for $y\omega$, which has been found necessary to represent the rapid convergence of the upper-state levels, the constants in this equation are very similar to those given by Butkow for the absorption bands alone. Although many more bands are studied in the present work, it does not follow



TlBr emission spectrum.

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Table 1. Vibrational analysis of sharp bands of main system

Italics denote wave-lengths in air.
Large Roman type, wave-numbers *in vacuo*.
Small Roman type, wave-number differences.
Numbers in parentheses, intensities.

v' \ v''	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	(10) 29149·9 *3429·56*	(9) 28958·2 *3452·26*	(6) 28767·4 *3475·17*	(4) 28578·2 *3498·17*	(3) 28389·4 * *3521·44*	(3) 28197·3 *3545·53*								
	69·3	95·6	97·6	97·1	96·1	98·4								
1	(4) 29246·2 *3418.27*	(1) 29053·8 *3440·91*	(4) 28865·0 *3463·42*	(5) 28675·3 *3486·32*	(5) 28485·5 * *3509·56*	(4) 28295·7 * *3533·10*	(3) 28109·4 *3556·52*	(2) 27922·9 *3580·27*	(1) 27737·3 *3604·23*					
	85·9	86·0						86·4	86·9					
2	(1) 29332·1 *3408·22*	(4) 29139·8 *3430·75*					†	(4) 28009·3 *3569·23*	(5) 27823·2 *3593·10*	(5) 27638·8 *3617·07*	(4) 27454·4 *3641·37*	(2) 27270·6 *3665·91*	(1) 27087·0 *3690·76*	(1) 26901·1 *3716·15*
		69								70	70	71	72	74
3	†	(3) 29209 *3422·6*	†	(1) 28830 *3467·6*	(1) 28640 *3490·6*				†	(2) 27709 *3607·9*	(3) 27525 *1632·3*	(4) 27341 *3656·4*	(4) 27159 *3681·0*	(5) 26976 *3705·9*

* Absorption measurement only. Overlaid in emission by Tl line.
† Band present, but too weak to be measured.

Table 2. Difference between observed and calculated wave-numbers ($\nu_{obs} - \nu_{calc}$)

v' \ v''	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	1·4	1·0	0·7	1·2	1·3	−2·6								
1	0·6	−0·7	1·0	1·0	0·1	−1·6	−0·6	−0·5	−0·4					
2	1·3	0·4						0·8	0·5	1·0	0·7	0·2	0·5	−3·6
3		−2		−1	−2					−1	−1	0	1	−2

Table 3. Br isotope effect in TlBr

v' \ v''	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	(0·3)	(2·1)	3·8 (3·8)	5·5 (5·6)	7·6 (7·2)	9·6 (8·6)								
1	(0·4)	(1·4)	2·5 (3·1)	4·8 (4·8)	7·0 (6·4)	7·3 (8·0)	9·7 (9·7)	10·4 (11·4)	10·9 (13·0)					
2	(1·0)	(0·7)						10·3 (10·9)	11·8 (12·5)	13·7 (14·1)	16·9 (15·5)	17·0 (17·3)	17·2 (18·8)	19·2 (20·0)
3										14·5 (13·8)	17 (15·4)	18 (17·0)	19 (19·0)	22 (20·1)

Calculated values in parentheses.

Wave-numbers of Tl ^{79}Br heads can be obtained by adding isotope separation to Tl ^{81}Br head given in table 1.

that the above constants are derived from more numerous data, because only those bands with well-defined heads have been used in the calculation, the greater part of the spectrum consisting of diffuse bands for which such mathematical representation is hardly warranted. For the sake of economy of space, these bands are not shown in table 1—they go as far as $v'' = 41$—but they are given in the list of classified bands in table 5. In view of the greater dispersion used here, 7·4 A./mm. at 3400 A. as against the 16 A./mm. used by Butkow, the constants given here are considered more reliable. The least certainty is attached to the values for the upper state, as only four vibrational levels have been used in the calculation. The agreement of the calculated wave-numbers of the heads with those observed can be examined in table 2.

Intensities of the bands have been assigned on a visual scale from 0 to 10, and are also given in table 1. The Franck-Condon parabola for these intensities is that which is to be expected for a molecule with the ω values so unequal.

§ 5. THE ISOTOPE EFFECT

(a) *Br*. Apart from those near the system origin, each band (sharp or diffuse) shows the doubling to be expected for the molecules Tl ^{79}Br and Tl ^{81}Br. For these, the splitting factor $(\rho - 1)$ is 0·009, and the agreement between the observed and theoretical splitting can be examined from table 3 for those bands for which accurate measurements have been possible. The separations for the diffuse bands can be estimated from the wave-numbers given in the list of classified bands in table 5. The largest splitting measured is 59 cm.$^{-1}$ for the 5, 41 band, for which the theoretical value is 56 cm.$^{-1}$. There does not therefore appear to be any unusual divergence from the theoretical values.

(b) *Tl*. Miescher (1939) has reported a doubling in some TlCl bands which he ascribes to the Tl isotopes ^{203}Tl and ^{205}Tl, although no data are given. In this case, with TlBr, $(\rho - 1)$ is 0·0007, i.e. about 13 times less than the value for the Br isotopes, and so the biggest splitting to be expected in the observed spectrum is of the order of 4·5 cm.$^{-1}$. In view of the diffuseness of the heads in this region this is about the order of uncertainty in the actual individual measurements, and so any possible resolution of the Tl isotopes can be ruled out.

§ 6. THE BANDS AT 3950 A.

There is a small group of sharp bands at 3950 A. which appears only in emission. The members of this system are much more intense than those of

Table 4. 3950 system

v' \ v''	0		1		2		3
0	25340·2 (10) *3945·18*	96·1	25244·1 (8) *3960·20*	86·3	25157·8 (6) *3973·80*	67·8	25090 (2) *3984·5*
	73·6		71				
1	25413·8 (8) *3933·75*	99	25315 (4) *3949·1*				

the main system. They are red-degraded and can be fitted into the accompanying fragmentary quantum array.

Other bands may belong to this system, but the presence of the diffuse bands and continuum makes their detection difficult and uncertain.

The lower state for these bands thus appears to be the same as the upper state for the main system. Although the isotope effect for this group is too small

Table 5. Classification of remaining TlBr bands

$Tl^{81}Br$		$Tl^{79}Br$		Separation	v', v''
λ	ν	λ	ν	$\Delta\nu$	
4493·8	22247	4505·6	22188	59	5, 41
4465·5	22388	obscured by Br lines			4, 40
4458·7	22422				5, 40
		"	22487 " "		
4435·3	22540	4445·7	22487	53	4, 39
4425·9	22588	4435·3	22540	48	5, 39
4403·0	22705	4412·4	22657	48	4, 38
4394·2	22750	4403·0	22705	45	5, 38
4371·6	22869	4380·4	22823	46	4, 37
4361·7	22920	4371·6	22869	51	5, 37
4338·9	23043	4347·8	22994	49	4, 36
4328·9	23094	4338·9	23043	51	5, 36
4306·9	23212	4315·7	23165	47	4, 35
4297·4	23863	4306·9	23212	51	5, 35
4274·7	23387	4283·6	23338	49	4, 34
4264·6	23442	4274·7	23387	55	5, 34
4245·7	23547	4254·6	23497	50	4, 33
4231·6	23625	4238·8	23585	40	5, 33
4214·4	23722	4223·2	23672	50	4, 32
4192·0	23848	4201·4	23796	52	3, 31
4185·0	23888	4192·0	23848	40	4, 31
4162·3	24018	4171·4	23966	52	3, 30
4154·8	24062	4162·3	24018	44	4, 30
4132·8	24190	4141·9	24147	43	3, 29
4102·4	24369	4109·7	24326	43	3, 28
4072·6	24547	4080·3	24501	46	3, 27
4044·6	24617	4053·2	24671	46	3, 26
4015·0	24900	4021·2	24861	39	3, 25
3986·7	25076	3992·8	25038	38	3, 24
3933·6	25415	3938·8	25381	34	3, 22
3907·2	25587	3911·7	25557	30	3, 21
3880·8	25761	3884·6	25735	26	3, 20
3861·5	25889	3865·7	25861	28	3, 19
3854·7	25935	3858·9	25907	28	4, 19
3834·1	26074	3838·6	26044	30	3, 18
3828·9	26110	3832·7	26084	26	4, 18
3779·6	26450	3783·6	26422	28	3, 16
3755·2	26620	3758·3	26600	20	3, 15
3730·9	26796	3734·3	26771	24	3, 14

In addition to the bands, the following lines have been measured on the plates:

4441·7, 4472·6, 4477·8, 4513·5 } Br and 4057, 3519, 3529, 3776 } Tl II, Tl

to be detected, and thus confirm that TlBr is the emitter, the following reasons suggest that this is so:—

(*a*) There is no evidence of the presence of other elements in the discharge, no lines other than those of Tl or Br being photographed.

(*b*) The absence of rotational structure indicates that the emitter is a heavy molecule such as TlBr or Tl_2, and with the latter the bands would probably be diffuse.

(*c*) It seems improbable that any foreign molecule would give bands of intensity so much greater than those of the main TlBr system.

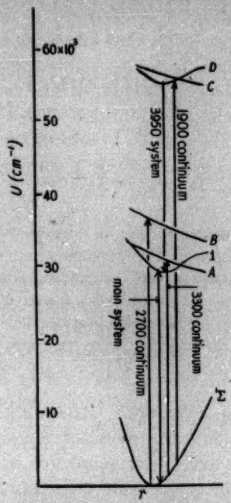
Finally, it will be shown that it is possible to account for the high intensity and the existence of only a few bands in this system on the assumption that TlBr is the emitter.

§7. PREDISSOCIATION; ENERGY LEVELS OF TLBR

The presence of a continuum on the short-wave end of the main band-systems of TlF and TlCl is accompanied by the occurrence of predissociation in these systems. In the present case, it has already been pointed out that no bands belonging to v' levels higher than 3 occur in absorption, and that in emission, although higher levels are involved, the bands are exceedingly diffuse, and even these are cut off abruptly at $v'=5$, an obvious indication that predissociation sets in near $v'=3$ and is complete at $v'=6$. Assuming that the point of intersection of the unstable level with the upper level of the band-system occurs for v' between 5 and 6, its energy has the value 29600 $cm.^{-1}$, which is in close agreement with 29620 (the red edge of the 3330 A. continuum), so that the upper state of the continuum can be safely identified as the one producing diffuseness and predissociation in the bands. Taking this red edge as representing transitions from $v''=0$ to this upper unstable level, the weaker continua at 3380 and 3430 A. most likely result from transitions from higher v'' levels to the same state. It will be noted, too, that since the upper state of the main band-system is the lower state of the 3950 system, the number of v'' levels of this system will be similarly limited.

In the figure a suggested energy-level diagram is presented. Considering the great similarity between the present spectrum and that of TlCl and TlF it is very probable that the corresponding levels and transitions are all of the same type. Thus the ground state may be taken as $^1\Sigma^+$ and the upper state of the main band-system as the component $\Omega=1$ of the complex triplet $\Omega=0^+$ and $^-$, 1, 2. The level A, which produces both predissociation and the continuum overlying the band-system, is found in both TlCl and TlF, and shows the expected lowering in energy for TlBr. The 1 state is the lower of the 3950-A. system, the upper one being D, which is probably crossed by the repulsive state C, producing predissociation and thus limiting the number of available v' levels for the 3950-A. system. Thus both upper and lower levels of this system are abbreviated by predissociation, which accounts for the small number of observed bands. If the level D is considered to be triplet in nature, then the transition $D\rightarrow 1$ will be much more intense than $1\rightarrow{}^1\Sigma^+$, because of the $\Delta S\doteq 0$ rule, which (although not operating rigorously here because of the absence of definite S values) will, nevertheless, give an indication of the probability of transition. Similar intensity considerations suggest that the upper state B of the continuum

at 2700 A. is singlet. The emission continuum at 2960 A. may involve another state in the region of B; and in TlF there is, indeed, such another level. On the other hand, these two continua may involve the same upper level, the extent and maxima merely differing in emission and absorption because of the difference in initial level population in the two cases. Besides, it is difficult otherwise to understand why the 2960 A. continuum is not observed in absorption.



Energy levels of TlBr.
The scale of r is quite arbitrary.

The position of level D can be located at $29200+25350=54550$, and the $D \rightleftarrows {}^1\Sigma$ transition should be found in both emission and absorption from about 1830 A. towards longer wave-lengths. All that is observed in the present investigation is the continuum stretching from 1900 A. onwards, resulting from the transition $C \leftarrow {}^1\Sigma$.

§ 9. ACKNOWLEDGEMENT

The authors wish to express their appreciation of the active experimental assistance and advice of Professor W. E. Curtis, F.R.S., during the progress of this work.

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AN ACHROMATIC REFLECTION MICROSCOPE FOR USE WITH VISIBLE OR ULTRA-VIOLET LIGHT

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Received 29 *July* 1941

ABSTRACT. This paper describes the performance of an achromatized optical system for a microscope for use over a wide range of wave-lengths either in the visible or ultra-violet spectrum, which arises from the continuation of a design initiated in 1934. An aluminized parabolic mirror and a LiF. SiO_2 objective are employed, and the calculated aberration values are given. Photomicrographs taken with the complete instrument (although unpublished) are available to any who may wish to see them.

In a paper in the *Journal of Scientific Instruments* (Johnson, 1934) the writer described an optical system for a reflection microscope which was designed and made both in glass and fused quartz for use in the visible and ultra-violet parts of the spectrum respectively. In this system, which employed a lens-mirror as one of the components of the objective, it was necessary to employ monochromatic light; nevertheless the instrument had some considerable advantages over the orthodox microscope inasmuch as the objective is much simpler in form and much easier to manufacture than the usually recognized type of microscope O.G., and secondly, when used for the examination of opaque objects, the optical system is entirely free from back reflections which frequently mar the image as seen with an ordinary objective used in conjunction with a vertical illuminator. A possible disadvantage of the system is that with numerical apertures exceeding 0·5 the aberrations become excessive, with a consequent deterioration in the definition of the image, but this defect is almost entirely offset by the fact that (because the instrument can be used with shorter wave-lengths) the effective numerical aperture can be increased to the equivalent of an oil-immersion objective used in visual light. At the same time, this form of microscope objective has the additional advantage that the depth of focus is two and a half times greater than that of the orthodox immersion lens. For example, the depth of focus in the object space is given by $\frac{\lambda/4}{n \,.\, \sin^2 U/2}$,

where U is the semi-angle taken in by the lens, and n the refractive index of the medium in the object space. For an immersion objective this angle may be as much as 60°, and for visual light this gives a depth of focus of 0·00037 mm. (or 0·37 μ), whereas with the reflection type of objective working in air at a numerical aperture of 0·5 (i.e. $U = 30°$) and at $\lambda = 2749$ A., the depth of focus is 1·0 μ.

It is probably still not generally realized, however, that the spherical aberration can be corrected and the coma kept down to quite small amounts when such a simple optical system as that described in the aforementioned paper is used. In order to give some idea of the excellent definition given by this instrument, photomicrographs * of various types of object (transparent and opaque) have recently been taken.

The writer felt, however, that the instrument would be improved if the optical system were made to be achromatic, so that one could use it with white

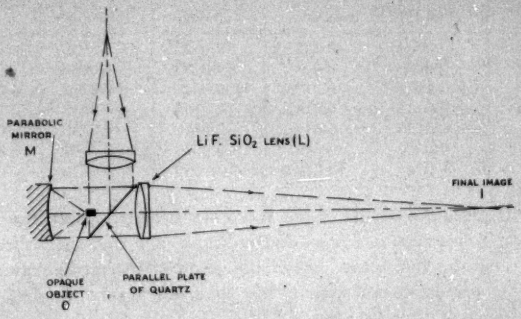

Figure 1.

light for visual work or with a range of wave-lengths in the ultra-violet. Matters relevant to this latter point, including the use of certain U.V. filters, are contained in a recent paper (Johnson, 1939), and do not therefore require repetition here.

To this end, a design was carried out which is shown diagrammatically in figure 1. The object O is situated at the focus of a parabolic aluminized surface M, and an achromatized doublet objective L made from lithium fluoride and fused quartz brings the parallel beam to a focus at I where the eyepiece observes the image. It is advisable to place the lens L at or near the centre of curvature of the mirror in order to reduce the astigmatism of the system to its minimum, but as the inclination of the beam to the axis is only three quarters of a degree (to fill the field taken in by the eyepiece), the location of L is not of first importance.

The computation of the objective L reveals some interesting facts; the first of these is that if wave-lengths 5461 A. and 2749 A. are brought to the same

* See footnote on p. 717.

focus, all other wave-lengths between λ 6000 A. and λ 2600 A. are corrected to within their permissible tolerances. Secondly, the spherical aberration is corrected (within the permissible tolerances) for each of these wave-lengths individually. Furthermore, the amount of coma present at three-quarters of a degree off axis for the objective alone is not in excess of its tolerance of ±0·0025. The employment of a parabolic reflecting surface, whilst not adding to the chromatic or spherical aberration, inevitably introduces additional coma; but calculations show, for example, that at λ 4048 A. and for N.A. = 0·47, the coma is only increased from 0·00170 to 0·00297 when using the parabolic mirror in conjunction with the lens system. This is certainly above the coma tolerance, and there would be a tendency to slight flare in the image at the extreme margin of the field, but this would not be serious.

Wave-length	Chromatic aberration (inches)	Chromatic aberration tolerance	Spherical aberration (inches)	Spherical aberration tolerance	Coma
5893 A.	−0·0024	±0·0235	+0·0287	±0·0940	+0·00149
5461	0·0000	±0·0214	+0·0243	±0·0856	+0·00156
4861	+0·0073	±0·0193	+0·0231	±0·0772	+0·00162
4048	+0·0132	±0·0162	+0·0156	±0·0648	+0·00170
3651	+0·0130	±0·0145	+0·0110	±0·0580	+0·00205
2749	0·0000	±0·0111	+0·0020	±0·0444	+0·00218
2573	−0·0141	±0·0104	0·0000	±0·0416	+0·00234

The table gives the results of the aberration calculations on the LiF. SiO_2 objective which has been designed, and indicates how well such a lens may be corrected; and also how suited the combination of these two materials is for removing the chromatic aberration, the spherical aberration and the coma over a wide range of wave-lengths.

The complete specification of the optical system is as follows:—

Parabolic mirror: Focus = 1·30″ Diameter = 1·25″
Radius = 2·60″ Numerical aperture = 0·47.

LiF. SiO_2 objective

		Axial Thickness
r_1 =	+ 4·667″	LiF lens = 0·20″
r_2 =	− 4·667″	SiO_2 lens = 0·10″
r_3 =	+41·700″	

Primary magnification of microscope = × 13.

Theoretical resolving power of instrument $= \frac{0{\cdot}61\,\lambda}{\text{N.A.}} = 0{\cdot}36$ microns.

Necessary total magnifying power to do justice to the resolving power = × 140 (for λ 2749 A.).

Both objective and mirror were made by the writer in the glass workshop of the Technical Optics Department of this College, and a word as to the method

employed for testing the figure of the parabolic surface may be of interest. The familiar Foucault knife-edge method was used, but in modified form so as to admit of the test being applied to a *short-focus* mirror. The scheme employed is depicted in figure 2; a minute hole in a piece of tinfoil (or silvered cover glass) is mounted at the focus F of the mirror. Light from a high-intensity source S is focused on this hole by means of a condenser and small right-angled prism (about $\frac{1}{8}''$ square face), thus giving an artificial star at F. A *well* corrected telescope objective (of, say, 16″ focus and $1\frac{1}{4}''$ diameter, or the LiF. SiO_2 lens already made) is placed at T and the knife-edge apparatus is situated at the focus of the latter. If the mirror M is correctly figured, all rays reflected by it will emerge parallel to the axis and be brought to a focus at K, when the knife-edge cutting the beam at this point will produce a uniform darkening of the field as observed by the eye. An additional advantage of this method is that by replacing the knife-edge by an eyepiece, the familiar star test may also be applied for judging the aberrations of the system as a whole. At a numerical aperture of 0·47, the

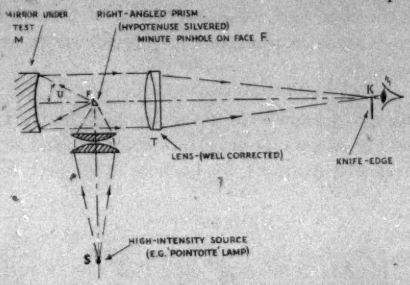

Figure 2.

departure of the parabolic curve from the true sphere amounts to less than 5/1000 inch increase in radius, so that for such apertures the figuring of the surface does not present undue difficulties.

One of the advantages of the achromatized nature of this optical system is that the object (once focused) may be illuminated by light of different wave-lengths and viewed or photographed without having to alter the fine adjustment of the microscope. As an illustration of this point, photomicrographs have been taken of a specimen of copper sulphide focused visually for λ 5461 A. and photographed at this and two other widely separated wave-lengths without further focusing of the instrument. As the spherical aberration is simultaneously corrected, it is in fact apparent from the photographs that the definition is still perfectly satisfactory. The differential reflection effects are also of particular interest in these three photographs.*

* The author regrets that it has not been possible to publish these and other photographs, but the Society did not (under present conditions) consider it would add to the usefulness of the paper by doing this. They are, however, available at the Imperial College to any who may wish to see them.

A second advantage is that the object may be illuminated by any desired ultra-violet wave-length and the fluorescent effects which frequently occur in many types of object may be observed with the optical system which is already corrected for visual light. Examples of this latter case have been photographed in colour.* These photomicrographs in colour are only intended to convey what the eye actually sees when looking into the microscope, and it is not suggested that such photographs should be taken for all specimens which fluoresce; the exposure required in this case, for example, was seventy-five hours, and thus the time would in general be prohibitive.

Apart from the use of the LiF . SiO_2 objective for microscope work, it will be obvious from the figures given in the table that a lens of this type would serve admirably as a telescope objective for visual light. The simultaneous correction of chromatic aberration, spherical aberration and coma should make the performance of the lens distinctly better than is obtainable with doublets made from hard crown and dense flint or from the barium crown and light flint glasses. Moreover, if the telescope and collimator lenses of ultra-violet spectrometers utilized such a lens as here described, no focusing difficulties would arise.

ACKNOWLEDGEMENTS

The writer wishes to acknowledge the advice given by Dr. C. H. Beale of the research department of Messrs. Dufay-Chromex, Ltd., and to Mr. J. McNeill, of Melbourne University, Australia, for his help in the processing of the colour photographs.

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DISCUSSION

MR. T. SMITH. There must be many who will be much interested to learn the extent to which aberrations and their chromatic variations can be kept within the desirable limits when materials transparent to ultra-violet light are combined to secure primary achromatism in both the visible and near ultra-violet parts of the spectrum. There are doubtless others besides myself who like to make calculations on other systems than that used by the author, and they would appreciate the addition of the refractive indices of both materials for the wave-lengths mentioned in the paper. I think some who content themselves with studying the results recorded in the paper would be assisted by the addition of a few more words of explanation. For instance; the sign conventions for the longitudinal aberrations (presumably the values given are longitudinal) can probably be guessed from those affixed to the chromatic aberration, but the sign and exact significance of the coma figures and of the tolerance, which is

* *Cf.* footnote on p. 717.

probably based on definite assumptions not mentioned in the paper, need explanation. Also, are the spherical aberration figures those of a marginal ray or of the ray showing maximum aberration, and are they all reckoned from the common paraxial focus for wave-lengths 5461 and 2749?

Mr. Johnson has shown by his recent papers that there are decided possibilities of constructing microscopes achromatized over a long spectral range, and we hope he will be able to continue this work with a view to the production of systems of large numerical aperture, so that full advantage can be taken of the short wave-lengths transmitted by such media as fused quartz and lithium fluoride.

AUTHOR'S REPLY. I thank Mr. Smith for raising some points which must necessarily be of importance to those who might wish to carry out calculations on lens systems which involve the use of lithium fluoride and fused quartz. The refractive indices of these two materials for some of the more generally used wave-lengths are as follows:—

Wave-length (A.)	Refractive index	
	Lithium fluoride	Fused quartz
5893	1·3920	1·4585
5461	1·3929	1·4602
4048	1·3983	1·4697
3651	1·4013	1·4751
2749	1·4128	1·4961
2313	1·4244	1·5194

The values given for spherical and chromatic aberration are longitudinal aberrations. They are for rays traced at $1/\sqrt{2}$ of the full aperture of the system. The chromatic aberration for other wave-lengths is expressed with respect to the common focus for λ5461 A. and 2749 A., whereas the spherical aberration is given as the difference between the intersection lengths of the marginal and paraxial rays, both traced for any *one* wave-length.

The last column of the table relates to the values of the "offence against the sine-condition", and gives the ratio of the coma-patch size to its distance from the centre of the field. The tolerance of 0·0025 is based on the fact that if we assume the useful limit to the angular field of view of eyepieces used with optical instruments as 13½ degrees, and a coma patch subtending a minimum perceptible angle of 2 minutes of arc to the eye (i.e. normal visual acuity), this latter (approximately 1/400 of the whole angle) gives a practical working basis for a ratio-value which should not in general be exceeded.

I am, indeed, glad to know that Mr. Smith is contemplating the possible use of lithium fluoride in the design of lens systems, and it is to be hoped that others will take advantage of the opportunity of developing new designs now that this and other synthetic optical crystals are gradually becoming available.

NOISE IN RECEIVING AERIAL SYSTEMS,
by R. E. BURGESS*

DISCUSSION

Mr. D. A. BELL. My treatment of radiation resistance from the point of view of Nyquist's theorem (Bell, 1939) was tentative and admittedly open to criticism; the discussion of it by the author was therefore a welcome as well as a very useful contribution to the problem, but it still leaves some difficulties on the theoretical side. First, it must be emphasized that calculated radiation resistances are "free-space" values, and an aerial in a uniform-temperature enclosure is obviously not in free space; therefore some evidence is required that the interchange of energy between the aerial and the uniform-temperature enclosure is a function of the free-space radiation resistance, and not of an additional mutual impedance between the aerial and enclosure. I believe this can be demonstrated.

Secondly, if an aerial is coupled to a material circuit as well as to free space, why should its radiation resistance, under equilibrium conditions, be at the temperature of free space rather than at that of the material circuit or somewhere between the two? The answer can only be obtained by examining the mechanism of the thermal fluctuations in an aerial.

Mr. Burgess says: "It is agreed that the work done on any current element (i.e. electron in motion) by the other electrons in the aerial will, on the average, be zero, for their field is randomly related to the motion of the electron considered. Work will be done, however, on each electron by its own field: on the classical theory, the power radiated by an electron with acceleration f is

$$P = \frac{2}{3}\frac{e^2 f^2}{c^3},$$

where e = electronic charge and c = velocity of light *in vacuo*." But there is no statement here of the nature of either the acceleration f or the frequency of the resultant radiation. If the accelerations in question are those due to the individual collisions between free electrons and molecules, the power radiated cannot be a function of radiation resistance; for the radiation resistance is a function of the geometry of the whole aerial, and the individual electron trajectories are so short as to be unaffected by this factor. Such radiation will be part of the thermal radiation of the material of which the aerial is constructed, not part of the energy loss represented by the radiation resistance of the aerial as a whole.

Examination of the mechanism of thermal-agitation e.m.f. in a metallic conductor (Bell, 1938) shows that the primary phenomenon is a displacement of charge, i.e. a current, made up of random components occurring at thermal frequencies; owing to the random nature of this displacement of charge, its average value over longer periods is not zero, and the impedance characteristics of the circuit operate upon it, in a manner which can be calculated by a Fourier

* See this volume, p. 293.

analysis, to produce components of radio-frequency current and voltage from the resultant of the higher-frequency electronic impulses. Now in an aerial in free space it is only the conductor-loss resistance which contributes to these thermal-frequency current pulses; but the total circuit impedance, including the radiation resistance, controls the resulting components of current and voltage at radio frequencies. If the circuit consists of a pure resistance shunted by a condenser, the mechanism of conversion of the thermal-frequency current pulses to a radio-frequency voltage fluctuation is easy to see: the thermal-frequency current pulses will transfer elements of charge into the condenser, and the resulting fluctuating voltages across the circuit will be a function both of the capacitance of the condenser (which governs the peak voltage attained) and the shunt resistance (which governs the rate of decay of charge on the condenser). In an inductive resistance, the magnetic field forms the integrating mechanism which similarly converts the thermal-frequency current pulses into radio-frequency voltages: just as the current pulses set up a voltage across the resistance-condenser circuit which decays according to the time-constant of the circuit, so in the inductive resistance the current pulses set up a resultant magnetic field which allows the current to change only according to the time-constant of the resistance and inductance of the circuit. In either case it is found by experiment that the phenomena can be represented by an equivalent circuit in which the resistance is separated from the reactance and has associated with it a generator of voltage fluctuations, whose magnitude is proportional to the resistance but independent of the reactances in the circuit.

A radiating aerial is closely analogous to a reactive resistance, for in each case the flow of current is associated with the establishment of electro-magnetic energy in the space near the circuit; the difference is that in a reactive resistance the energy is merely stored for a half-cycle, whereas in a radiating aerial the energy is permanently lost from the circuit. Consequently, the primary fluctuations generated in an aerial circuit are a function of the local-loss resistance only, not of the radiation resistance, but the radio-frequency current and voltage distribution are functions of the total impedance of the circuit. It follows that my earlier suggestion (Bell, 1939), that the aerial would not radiate energy derived from the thermal agitation in the metallic resistance, was incorrect; for although the individual current pulses do not radiate at radio frequency in the manner implied by Burgess, these pulses are integrated by the circuit, and the resultant fluctuation currents, which are a function of the whole circuit, do radiate in proportion as the aerial has appreciable radiation resistance at their particular frequencies. The author's method of calculating noise in an aerial in free space is then correct: in calculating the noise of an aerial in free space, the radiation resistance is to be counted as a resistance at zero temperature, and the local-loss resistance is a source of fluctuation voltage corresponding to its own temperature.

The idea of an " equivalent temperature of radiation resistance " of an aerial which is not in free space, however, may prove misleading when the noise is not entirely of thermal origin.

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AUTHOR'S REPLY. A uniform temperature enclosure is usually considered to have " perfectly black " walls whose absorptive power is unity at every frequency, and in such an enclosure the radiation from an aerial would be completely absorbed, so that the aerial impedance should have the same value as in free space.

In considering the resistance of an aerial, it can be stated that the power expended in the aerial will ultimately be either

(*a*) dissipated in material circuits to which the aerial is coupled, in which case the corresponding components of resistance must be considered as having the same respective temperatures as the circuits; or

(*b*) radiated into free space (perhaps after re-radiation by other circuits) without further loss, the corresponding component of radiation resistance being effectively at zero temperature.

I am essentially in agreement with the discussion of the process by which the thermal agitation of the electrons gives rise to radio-frequency radiation, although it might better be termed one of *selection* than of *conversion*. This discussion may be regarded as a detailed paraphrase of the first two equations on p. 295 of the original paper, which express the mean square fluctuation current and mean radiated power in terms of the R, R_r, and Z of the aerial.

Fundamentally, the acceleration of the electrons must, on the classical theory, be the source of radiation, and I think it is possible to show generally that the relation $d\overline{P_{\text{rad}}} = R_r d\overline{i^2}$ must hold when R_r and $d\overline{i^2}$ are measured at the same pair of terminals. A corroboration of this view is to be found in two recent papers by Fränz (1940) and Thompson, North and Harris (1941), where the same conditions for the thermal equilibrium of an aerial in a uniform temperature enclosure are derived.

I agree that the " equivalent temperature of radiation resistance " would be misleading if incorrectly used, but its only significance is an *equivalence* which considerably facilitates the computation of noise in aerial circuits, as was demonstrated in the original paper.

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OBITUARY NOTICES

GEORGE WILLIAM CLARKSON KAYE

BORN at Harrogate 8 April 1880, the son of an architect, Kaye must have shown an interest in physics at quite an early age, for he has told the writer how the science master at his school set up an x-ray tube as soon as he heard of Röntgen's discovery, permitting the boy to come in at the week end to help in this exciting work. In 1899 Kaye came to London to attend the Royal College of Science, taking its Associateship in 1902, with first-class honours in physics. Here he attended the lectures, among others, of Callendar (who was appointed in 1901) and of John Perry. After taking his degree he remained as demonstrator, and was in 1904 appointed a teaching associate. It was during this period that he met the daughter of Professor Willis, then a girl student of physics, whom he subsequently married.

In 1905, Kaye went to Cambridge, taking his B.A. in 1908. His research in the Cavendish was on the subject that had occupied those Sunday mornings at school—x rays. Among other work, he investigated the variation of ionization produced by the x rays from targets of different materials (it was too early to deal with wave-lengths) and found that it was closely proportional to the atomic weight. At Cambridge he made the acquaintance of many men well known in the field of physics, among them his approximate contemporaries J. A. Crowther (whose notice of Kaye appears below) and T. H. Laby, with whom he began the project that led to Kaye and Laby's Tables, of which the ninth edition was in preparation at the time of his death. But above all his contemporaries he felt an admiration and affection for " J. J.", whose personal assistant he became.

The work which Kaye carried out at Cambridge was in some ways the best of his career. He was at heart an experimentalist, and though he did not despise theory, he had an impatience with it, and could always find more pleasure in a neat experimental proof than in a logical deduction. His interest in the manipulative side of the physicist's craft is well illustrated by the practical hints in the Appendix to his book on x rays, and was again forcibly brought out when, in 1937, as President of Section A of the British Association, he gave a Presidential address copiously illustrated with experiments, and delighted in the fact that he was setting such a precedent.

In 1910 he left Cambridge to join the staff of the National Physical Laboratory, where one of his first jobs was in connection with the proposed silica standard of length. This appealed to his temperament, and he wrote a most interesting paper on the matter. On the transfer of the thermometry testing from Kew Observatory to the N.P.L., Kaye was put in charge of this work.

But shortly after this came the first World War: Kaye was already in the London Electrical Engineers (Territorials), and on the outbreak of war proceeded with his unit to the Thames Estuary where he did searchlight work, a technical job much to his liking. Later he transferred to the Aeronautical Inspection Directorate, which involved his travelling to Canada and the United States of America in connection with the purchase of timber; from 1917 to 1920 he was chief inspector of materials, Air Ministry, with the rank of Major. He later received the O.B.E. for these services.

On his return to the N.P.L. after the war, he took over the Physics Department, which then consisted of a number of separate divisions dealing respectively with heat, thermometry, optics and radiology. The administrative aspects of the work took more and more of his time, and he had little opportunity for research, though he continued to carry out some with the aid of assistants, not only in his own subject of radiology, but also in thermal conductivity and high vacua. The well-known Kaye pump was designed at this period. At the same time he initiated and built up a flourishing acoustics division, concerned mainly with such practical aspects as architectural acoustics and the noise nuisance. This division grew so large that it must have been a relief to him when, in 1940, the optics division was separated from the Physics department and was merged with the photometry division of the Electricity department, to form a new department of Light.

In the last twenty years his time was mainly occupied with administrative and committee work, where his command of English and patience in re-drafting until perfection was approached were of inestimable value. He had turned his attention to the physical aspects of medical radiology, especially to x-ray and radium dosage and protection. His untiring work for the establishment of the National X-ray and Radium Protection Committee, and his continuous active service for it, led naturally to his being a delegate to each of the triennial International Congresses of Radiology from their inception in 1925 and secretary of the International X-ray and Radium Protection Commission from 1927 onwards. He was also a member of the National Radium Commission, Chairman of the Ministry of Transport Committee on Acoustics and Noise, and a member of numerous other committees; at one time, for example, he was concerned with no less than ten committees of the British Standards Institution. Among all these activities, he found time to write, in addition to the earlier book of Tables, and that on x rays, one on high vacua, one entitled *Practical Applications of X rays*, one entitled *Roentgenology*, and a joint work with A. H. Davis on *Architectural Acoustics*.

Some five years ago it was evident to those in close contact with him that he was becoming a tired man. He was more reluctant to take big decisions, and he had spells of illness from time to time. In July 1940 he developed general toxaemia and, though he continued to direct the affairs of his department from his sick room (he was not entirely confined to bed), he never recovered, and the tired heart stopped working on 16 April 1941, eight days after his birthday. During

this illness he had had the happier diversion of seeing his elder daughter and his only son married.

His work had not been without recognition. He had been President of Section A of the British Association, of the Röntgen Society and of the British Institute of Radiology; was President of the Society of Radiographers and an honorary member of the American Roentgen Ray Society, to whom he delivered the Caldwell lecture, and an honorary member of the corresponding Scandinavian body. He had twice given the Cantor lectures and once the Aldred lecture at the Royal Society of Arts; also the Tyndall lecture at the Royal Institution, the McKenzie Davidson and the Silvanus Thompson lectures of the British Institute of Radiology. He was elected F.R.S. in 1939.

He loved to talk of past days, and was never happier than when he could lunch at the Athenaeum, of which he was a member, or relax at the Physical Society Club dinners. Here, as in his department, he was always willing to befriend younger colleagues, and to give them the benefit of his own wide experience. Other events to which he looked forward each year were the Cavendish dinner and the meeting of the British Association, particularly as they gave him the opportunity to meet old acquaintances and talk over past days.

He leaves a widow and one daughter in her teens, in addition to the two recently married children. J. H. A.

It is a privilege to be allowed to add a few personal recollections to the obituary notice which appears above. When I graduated into the research department of the Cavendish Laboratory, in 1905, Kaye had just arrived to take on the responsible duties of research assistant to "J. J." He, of course, worked in the Professor's laboratory, I in a sort of draughty passage leading directly out of it. As the door was never shut, we naturally saw a good deal of each other, and propinquity, aided perhaps by the fact that we both came from the same Riding of the same county (and were proud of it), quickly ripened into friendship. We were a happy little group; Satterly and Laby, I think, were there from the beginning, and we were soon afterwards joined by Whiddington, Thirkill, Paine and others. We worked hard. Kaye, in particular, carried a double burden, as he not only had to assist the Professor in his researches (no light task when "J. J." was hot on the trail of something new), but at the same time he was carrying on experimental work of his own with a view to taking his Cambridge B.A. by research. Kaye was a rapid worker with a real flair for experimentation, and these "part-time" researches on the production and distribution of x radiation are as good as anything he subsequently produced.

Somehow or other work did not seem seriously to interfere with other activities. It was often possible to slip out of the lab. for a strenuous game of fives, returning, heated but happy, in time to meet "J. J." (who could be relied upon to see the University Rugger match through to its conclusion). In the evenings there were meetings in each others' rooms for the discussion—among

other matters—of current scientific discoveries; and there was the great event of the year, the Cavendish dinner, which provided an excuse for numerous preliminary meetings and rehearsals. Kaye was at his best in all these activities. He liked the society of his fellow men and, although he never monopolized the centre of the stage, things somehow seemed to go more smoothly if he were present. At any Cavendish sing-song (and in those " pre-wireless " days we were fond of making cheerful noises) it was inevitable that Kaye should be called upon for *The Buffalo Battery*, a song of unknown origin with a rollicking chorus, which he rendered with an artistry which remains as a very pleasant memory. Kaye had dramatic talent well above the average, and the very last enterprise which I shared with him, only a year or two ago, was an educational Mock Trial, staged for the benefit of the Society of Radiographers, a society in which he took a keen and almost paternal interest, and of which he was President at the time of his death.

It happened that, except at the annual Cavendish dinner, which he rarely missed, I saw very little of him between the time when he left Cambridge for the N.P.L. in 1910 and a year or two after the last war, when we became associated again on the Council of the Röntgen Society, of which Kaye was one of the leading spirits. It would, I think, be difficult to exaggerate the interest which he felt in the subject of medical radiology, or the time, thought and energy which he devoted to its furtherance. His successful efforts, first to define and subsequently to secure the adoption of, adequate standards of protection for x-ray and radium workers, have been mentioned in the preceding memoir; the task of introducing some kind of order and precision into the field of x-ray and radium " dosage ", in which he also played a leading part, must have taxed even his patience and native doggedness before agreement was ultimately reached. It was fortunate that his position in the N.P.L. enabled him to enlist the resources of that great laboratory in the work.

But Kaye's interest in the progress of medical radiology was far from being limited to the assistance which he was able to bring to it as a professional physicist. Only those who have been in some measure behind the scenes can estimate the extent of his services to the cause. He was one of the prime movers in securing the amalgamation of the Röntgen Society with the purely medical radiological society (the name of which I have forgotten, but which was generally, I believe, alluded to as the B.A.R.P.) to form the now flourishing British Institute of Radiology. The welding of medical practitioners, physicists and x-ray engineers into a single society, and the close co-operation between them which was thus secured, have been important factors in the progress of British radiology. The amalgamation was not accomplished without difficulty, nor did the new Institute operate entirely without friction during its earlier years. It was due not a little to Kaye's tact and commonsense, and above all to his friendliness, that the Institute sailed at last into smooth waters. It was impossible to quarrel with Kaye; and it was almost impossible to quarrel with anyone else if Kaye were present. He was a

member of the Council, I believe, without interruption from its inception to the time of his death, and no one was more welcome there. For some years he edited the Journal. Latterly he took up the cause of the radiographers, the men and women who do the routine work of the radiological departments, and though often obviously a sick man, he gave unstintedly of his time and energy to their Society. I think that, during the later years of his life, Kaye was at his happiest at our radiological Conferences, Councils and meetings. The affection with which he was greeted and the unique position which he occupied among us were his well-deserved rewards. He will be greatly missed.

J. A. CROWTHER.

GWILYM OWEN

GWILYM OWEN, Professor of Physics and sometime Vice-Principal and Acting-Principal of the University College of Wales, Aberystwyth, was born at Denbigh, North Wales, on 19 July 1880, the son of the Rev. William Owen, and was descended from a family famous in Welsh life. He was educated at Ruthin School and at the University of Liverpool, where he was a pupil of the late Sir Oliver Lodge. In 1901 he entered Christ's College, Cambridge, as 1851 Exhibition Student, and pursued research under Sir J. J. Thomson. In 1905 he joined the staff of Professor L. R. Wilberforce at Liverpool University, and during the subsequent years established a reputation both as an experimenter of the first rank and as a teacher of unusual gifts. In 1913 he was appointed Professor of Physics at Auckland University College, New Zealand, but after two years he joined the New Zealand Expeditionary Force and left for active service in France, where he took a considerable part in the development of sound-ranging technique. He was twice wounded, but remained on active service until the end of the war, when he returned to New Zealand. In 1919 he was appointed Professor of Physics at Aberystwyth, in succession to the late Professor Morgan Lewis, who had held the chair since 1891, and was immediately faced with the stupendous task of reorganizing what had previously been a small department, both in numbers and in the extent of its equipment and accommodation, to deal with the large and sudden influx of students following demobilisation. Into this task he put his whole energy, and the present form of the department is a monument to his labours in the post-war period. He occupied various administrative positions in the College, and in 1932 was appointed Vice-Principal. In 1934, on the retirement of the late Sir Henry Stuart Jones, he became Acting-Principal for one year, and thereafter remained as Vice-Principal until 1936. In September of that year, whilst on a fishing holiday, he was suddenly struck by a severe illness, and was forced entirely to abandon his work at the College. It appeared at one time that his illness would respond to treatment, but in spite of all endeavours his condition grew steadily worse, and he formally resigned his appointments at the College in 1938. After four tragic years of ill-health, he died on 9 November 1940, and was laid to rest in Anfield Cemetery, Liverpool.

He was unmarried, and his only surviving near relative is his sister, to whom he was very devoted and whose home he made his own.

Whilst Professor Owen carried out a number of notable researches, first at Cambridge on thermionic emission in the days when electrons were still something of a novelty, and later, at Liverpool, on various kinds of condensation nuclei, he will be remembered chiefly on account of his superb quality as a teacher of physics. In his uncanny ability to appreciate and clear up the difficulties of the student, he can have had few equals, and his lectures were no ordinary lectures. He was a lecturer in the tradition of John Tyndall, and he had that rare and indefinable gift of making the dry bones live, so that his audiences, even when made up of people with no scientific training or background, were fascinated. His lectures were notable not only on account of the freshness and clarity of his exposition, but also in virtue of the beauty and ingenuity of the demonstrations and experiments by which they were illustrated. He spared no pains in the preparation of even the simplest experiment, and it was his invariable rule never to show an experiment at a lecture unless it was absolutely reliable and unless the particular effect it was designed to show was abundantly obvious. He was fortunate in possessing a very high degree of manipulative skill, the product of a natural dexterity developed by his Cavendish Laboratory training : even among Cavendish men of his generation, who had to rely almost entirely on their own manual ability, he was outstanding, his capabilities as a glass-worker, for example, being far above those of the average amateur.

It was inevitable that a lecturer of his quality should be greatly in demand outside his department. Within the college itself he gave a large number of lectures from time to time to various college societies, and his reputation as a master of his craft was such that the lecture-room was invariably packed to the doors with an audience which listened with rapt attention and greeted the end of the lecture with thunderous applause. Outside the college he was well known throughout Wales as a popular lecturer, and until the later years, when administrative work made great demands upon his time, he had each winter a heavy programme of lectures in the towns and villages of his native country. The great majority of these lectures were given in Welsh, and one of his great services to his generation was his presentation of scientific ideas in that language. He not only lectured in Welsh, but also wrote a number of books on various aspects of physics in Welsh, books which, to many Welsh people, opened a door to a department of knowledge hitherto strange to them. The last of these books was published in 1936, a few months before his breakdown, and was an elaboration of a series of radio talks given to schools in 1935. To one long accustomed to illustrate lectures with experiments, the technique of the microphone came strangely, but his essential directness and simplicity, allied to an excellent speaking voice, were well suited to the new medium, and his broadcasts were a great success.

In private life, he was a man of wide humanity and a genial companion. He was at his best in the society of intimate friends, around the fireside, playing

golf, or fishing, at both of which pastimes he was an expert: on such occasions he cast aside the somewhat solemn cloak which, in his professional capacity, he often appeared to wear, and which gave, to those who knew him only from afar, a wholly inaccurate impression of his real self. He was fundamentally a humble gentleman, with his roots deep in the traditions and culture of his people, and with the life of the Welsh countryside as his essential background. In his death, Wales has lost a gifted and faithful son.

I. C. JONES.

RICHARD THOMAS BEATTY

BEATTY was born at Dungannon on 26 April 1882 and educated at the Royal School of that city, and then, from 1901 to 1906, at the Queen's University, Belfast. He later worked as a research student at the Cavendish Laboratory, where he investigated the properties of homogeneous (characteristic) x rays. His work on the energy of the secondary electrons excited by these rays was of value in relation to the quantum theory, which was then beginning to take shape.

His first appointment was as assistant lecturer in physics at the University of Sheffield, which he left in 1914 on being appointed a lecturer at the Queen's University. During the war of 1914–1918 he served in a temporary capacity in the Admiralty Establishment at Portsmouth, became a member of the permanent Scientific Staff of the Admiralty in 1920, and remained there until his death.

Since 1920, he had published little, probably because of the secret nature of much of his work. It may be surmised that it was connected with such matters as the location of objects by echoes and depth sounding, for he made himself a recognized authority on acoustical subjects. His own interest appears to have been in physiological acoustics, and one of his last contributions to the *Proceedings* of this Society was during the discussion on *Audition*. This contribution was subsequently expanded into the book *Hearing in Man and Animals*, a standard work, in which the evolution of the organs of hearing is discussed, and rival theories of audition examined critically. His interest in music was artistic as well as scientific, and he enjoyed taking part in a quartet. He was also interested in radio and published a set of very useful *Radio Data Charts*.

Beatty died on 2 April 1941 and leaves a widow and two sons, one in the Army and one in the Navy.

In some ways rather reserved, yet a man of direct and fearless speech, Beatty's personality was an attractive one, and he will be missed by many besides his immediate friends.

REVIEWS OF BOOKS

Fourier Series and Boundary Value Problems, by RUEL V. CHURCHILL. Pp. ix + 206. (London : McGraw-Hill Publishing Co., Ltd., 1941.) 17*s.* 6*d.*

Although in books we sometimes find chapters on the numerical solution of differential equations, the title is a misnomer, since what is found is a solution of a differential equation subject to certain initial conditions. This aggregate of equation and initial (or maybe terminal) conditions is what is usually needed in physics, and is known to mathematicians as a boundary-value problem. It is this which can in some cases be found by operational methods, and it is this to which an integral equation is often equivalent. Frequently, the boundary-value problem can be solved when the general solution of the differential equation is unknown, or is useless for practical applications.

In 1822, Fourier, following Bernoulli and Euler and others, solved a number of problems of this nature, and gave a more general exposition of the theory than they had done. His work was understood to be of use in the solution of physical problems, but many years elapsed before it was seen that the field which he had opened up was so important as in fact it is. To-day, Fourier series and theory of functions of a real variable are almost co-extensive. In the modern, purely mathematical developments of the subject, a Fourier series is associated with a function, and the exceptional conditions under which the series represents the function, or has the same numerical values, are investigated afterwards.

In physics, however, the importance of Fourier series is that they represent functions numerically. The inner reason for this is that $\sin mx$ and $\sin nx$ are orthogonal functions, i.e. the integral of $\sin mx \sin nx$ over a suitable interval (in this case π) is zero if m and n are unequal. Thus if $f(x)=\Sigma A_m \sin mx$, and we multiply by $\sin nx$ and integrate from 0 to π, all terms but one of the series vanish, and we get A_m explicitly. (It is evident that the integral cannot vanish for $m=n$, since in that case the integrand is a square and cannot therefore be negative). Other orthogonal functions have been known for some time, e.g. the Legendre polynomials, but it is only in relatively recent years that the general study of them has been undertaken.

There is one other aspect in which Fourier's work has been fruitful, and that is the concept of the *Fourier transform*, which, however, lies away from the path taken by the book under review. This work is intended primarily for the physicist, and does not explore the purely mathematical developments, though the author is careful to state all his theorems in accordance with the requirements of mathematical rigour. There are two introductory chapters on differential equations and one on orthogonal functions. The next two deal directly with Fourier series, and chapter six exemplifies their use in the solutions of physical problems. The next chapter is on existence theorems, and the last two give elementary (but again rigorous) accounts of Bessel functions and Legendre polynomials.

The book is plentifully supplied with examples which should be within the power of an earnest student, and is beautifully produced and printed.

The author promises soon to publish a more advanced book on further methods of solving boundary-value problems, but gives no indication of the scope which he intends it to have. Whether it treats of the Sturm-Liouville problem, Laguerre polynomials, complex functions or operational methods, it seems likely from this foretaste that it will be interesting and useful.

J. H. A.

The Physical Examination of Metals, by BRUCE CHALMERS and A. G. QUARRELL. (Vol. 2, Electrical Methods.) Pp. viii+280. (London: Edward Arnold and Co., 1941.) 20*s*.

Following a brief introduction, there are six chapters under the headings *magnetism*, *electrical measurements*, *x-ray diffraction*, *diffraction of electrons*, *the electron microscope* and *radiography*. The chapter on magnetism begins with an account of those magnetic properties of matter which are used in testing materials ; the hysteresis loop, magnetization and demagnetization curves and variation of permeability with temperature are included. A variety of methods of measuring these properties is then described and the development of special methods for particular purposes, such as the determination of the carbon content of steels, receives attention. Methods are described for the detection of flaws in the surface of steel, for measuring the thickness of deposited coatings and the thickness of sheet or strip metal. The chapter on electrical methods of testing is chiefly concerned with a description of the standard methods of measuring resistance and the application of thermoelectric measurements in thermal analysis. The use of current distribution for measuring the thickness of plates which are accessible from one side only is described. The piezo-electric effect, the capacity micrometer and photo-electric cell are briefly treated. The next two chapters on x-ray and electron diffraction deal with tests on a much smaller scale of fine-grainedness. The fundamental notions concerning the architecture of crystals and the diffraction of waves by a three-dimensional grating form an introduction to these subjects. Examples of the sort of information to be obtained by x-ray crystal analysis are drawn from the structure of alloys, super-lattice formation and order-disorder transformations. The application of the method to the study of intermetallic compounds, the effect of deformation and the determination of crystal size are described. The advantages of electron diffraction as a method of investigating the structure of very thin layers on the surface, such as electro-deposits and oxide films, is stressed. The two concluding chapters on the electron microscope and radiography are very brief.

The very extensive range of subjects covered makes it inevitable that the descriptive matter should be superficial. To the reader who requires a bird's eye view of territory with which he is not familiar, the book may be recommended ; it could, however, be greatly improved by the inclusion of further references to sources of more detailed information, e.g. in the section devoted to *bridge methods* (pp. 82–8) reference to some standard text-book where fuller descriptions are to be found would be helpful. In a book of this type it is difficult to combine accuracy with brevity, but even after allowance has been made for the task the authors have set themselves, it must be said that the text contains more than a fair share of gratuitous misinformation. Careful revision would have eliminated some errors, e.g. in plate 2 "left to right" is incorrect, and figure 34 does not seem to agree with the description in the text. These are minor matters, as also are the statement on p. 152 that crystal sizes down to 10^{-8} cm. can be measured, and the equality (p. 236) of 100 A. and 10^{-7} mm. The statement (p. 139) that characteristic x-rays were unknown when the first Laue photograph was made is wrong. On p. 180 the symbol m is wrongly defined. On p. 2 it is implied that the whole of the energy of an electron is converted to x-rays on the anode. Are the lines of force correctly sketched in figures 20 and 22 (*a*), and why is the reader deprived of the promised values of μ in table 1 ?

G. D. P.

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